Abstract—This paper presents a model for microgrid optimal scheduling considering multi-period islanding constraints. The objective of the problem is to minimize the microgrid total operation cost which comprises the generation cost of local resources and cost of energy purchase from the main grid. The microgrid optimal scheduling problem is decomposed into a grid-connected operation master problem and an islanded operation subproblem. The microgrid capability in operating in islanded mode for multiple hours is scrutinized by a $T$-$\tau$ islanding criterion. The integer scheduling decisions determined in the master problem will be examined against the microgrid islanding feasibility in the subproblem. The scheduling decisions will be revised using proper islanding cuts if sufficient generation is not available to guarantee a feasible islanding. Islanding cuts include generating units, energy storage systems and adjustable loads schedules. Any change in the schedule of adjustable loads outside the operating time interval specified by the consumer is penalized by an inconvenience factor in the objective. Numerical simulations demonstrate the effectiveness of the proposed microgrid optimal scheduling model and explore its economic and reliability merits.

Index Terms—Microgrid optimal scheduling, islanded operation, distributed energy resource, adjustable load

NOMENCLATURE

Indices:
- $b$ Index for energy storage systems
- $ch$ Superscript for energy storage system charging mode
- $d$ Index for loads
- $dch$ Superscript for energy storage system discharging mode
- $i$ Index for DERs
- $s$ Index for scenarios
- $t$ Index for time
- $\wedge$ Index for calculated variables

Sets:
- $D$ Set of adjustable loads
- $G$ Set of dispatchable units
- $S$ Set of energy storage systems

Parameters:
- $DR$ Ramp down rate
- $DT$ Minimum down time
- $E$ Load total required energy

Variables:
- $F(\cdot)$ Generation cost
- $K_d$ Inconvenience penalty factor
- $MC$ Minimum charging time
- $MD$ Minimum discharging time
- $MU$ Minimum operating time
- $U$ Outage state of the main grid line/Islanding state
- $UR$ Ramp up rate
- $UT$ Minimum up time
- $\alpha, \beta$ Specified start and end times of adjustable loads
- $\rho$ Market price

I. INTRODUCTION

MICROGRIDS are introduced to address the emergence of a large number of distributed energy resources (DERs) in distribution systems and further address ongoing energy, economics and environmental challenges by making smarter power grids. A microgrid, which is technically a small scale power system with ability of self-supply and islanding, provides a distributed local intelligence for the power system to supply loads in a reliable and economic manner [1]-[5].

Microgrids introduce unique opportunities in power system operation and planning, such as improved reliability by introducing self-healing at the local distribution network and
lowering the possibility of load shedding, higher power quality by managing local loads, reduction in carbon emission by the diversification of energy sources, economic operation by reducing transmission and distribution costs and utilization of less costly renewable energy sources, offering energy efficiency by responding to real-time market prices, reducing the total system expansion cost by deferring investments on new generation and transmission facilities, and providing a quick and efficient response for supplying load in remote areas [6]-[13]. The salient feature of a microgrid is its ability to be islanded from the main power distribution network. Islanding is typically performed to rapidly disconnect the microgrid from a faulted distribution network to safeguard the microgrid components from upstream disturbances and allow an uninterrupted supply of loads. It is also performed to protect voltage sensitive loads from significant voltage drops when a quick solution to main grid voltage problems is not imminent [14]. The microgrid is economically operated in grid-connected mode, however, sufficient capacity should always be available in a case that microgrid is required to switch to the islanded mode. The microgrid is islanded from the main grid using upstream switches at the point of common coupling (PCC), and the microgrid load is fully supplied using local resources [10]-[12].

The microgrid scheduling in grid-connected and islanded modes is performed by the microgrid master controller based on security and economic considerations. The master controller determines the microgrid interaction with the main grid, the decision to switch between grid-connected and islanded modes, and optimal operation of local resources. The microgrid optimal scheduling performed by the microgrid master controller is considerably different from the unit commitment (UC) problem solved by an ISO for the main grid. Variable generation resources and energy storage systems have major roles in microgrid operation due to their considerable size compared to local loads. In addition, generation resources are close to load premises and power is transmitted over medium or low voltage distribution networks, hence the congestion would not be an issue in power transfer. A high percentage of local loads could also be responsive to price variations, which makes the microgrid load/generation balance more flexible. Finally, the connection to the main grid in grid-connected mode, which represents the main grid as an infinite bus with unlimited power supply/demand, enables mitigating power mismatches in the microgrid by power transfer from the main grid. The main grid could further provide reserve for the microgrid when the predicted variable generations are not materialized or load forecast errors are high. However, the optimal microgrid scheduling and the UC problem in the main grid share a common objective, i.e., to determine the least cost operation of available resources to supply forecasted loads while taking prevailing operational constraints into consideration. Although sharing a common objective, the mentioned differences would not allow a direct application of existing UC methods to the microgrid optimal scheduling problem. The rapid development of microgrids calls for new methodologies to comprehensively model all the active components in microgrids and particularly focus on microgrid islanding requirements when the main grid power is not available.

The microgrid optimal scheduling is extensively investigated in the literature. The existing energy management system architectures for microgrids are reviewed in [15], where centralized and distributed models are identified as common microgrid control schemes. The centralized model collects all the required information for the microgrid scheduling and performs a centralized operation and control [16]-[20]. In the distributed model, however, each component is considered as an agent with the ability of discrete decision making. The optimal schedule is obtained using iterative data transfers among agents [21]-[23]. Both control schemes offer benefits and drawbacks, but the centralized model is more desirable as it ensures a secure microgrid operation and is more suitable for application of optimization techniques. The main drawbacks of the centralized scheme are reduced flexibility in adding new components and extensive computational requirements [24]. These disadvantages are mitigated by the proposed model in this paper.

The microgrid islanding studies are very limited in the literature. [25] proposes an economic dispatch model for a microgrid which applies additional reserve constraints to enable islanding. [26] presents a load management model to improve microgrid resilience following islanding, taking into account the microgrid limited energy storage capability and frequency response. A method to determine the amount of storage required to meet reliability targets and guarantee on island-capable operation with variable generation is proposed in [27]. In [28] storage systems are applied in microgrids to balance power, smooth out load, reduce power exchange with the main grid in the grid-connected mode and ensure successful transition to the islanded mode.

This paper presents a centralized microgrid optimal scheduling model which considers multi-period islanding constraints. The objective is to minimize the day-ahead grid-connected operation cost of the microgrid using available generation resources, energy storage systems, adjustable loads and the main grid power, subject to prevailing operational constraints. The solution is examined for islanding to ensure the microgrid has sufficient online capacity for quickly switching to the islanded mode if required. An islanding criterion is proposed which demonstrates the resiliency of the microgrid to operate in islanded mode for a variety of time durations. An iterative model based on the Benders decomposition is employed to couple grid-connected operation (as a master problem) and islanded operation (as a subproblem). The iterative model significantly reduces the problem computation burdens and enables a quick solution. Problems are modeled using mixed integer programming which facilitates addition of new components to the microgrid.

The proposed model in this paper is developed specifically for microgrids. The novel contribution of the proposed model is to effectively consider uncertain microgrid islanding (from islanding time and duration standpoints) in the microgrid optimal scheduling problem. The proposed model enables the microgrid to operate in the islanded mode and adequately supply the local loads when the time and extent of the main grid disturbance is unknown. The islanding duration is considered via a novel criterion. A multi-period islanding is...
considered in this paper which refers to the islanding event that takes several hours long. The proposed model is comprehensive yet flexible in adding new components to the microgrid and benefits from a decomposed model that reduces computation burdens and makes it suitably applicable to centralized microgrid scheduling schemes.

The rest of the paper is organized as follows. Section II outlines the microgrid optimal scheduling model, introduces microgrid components associated with optimal scheduling, and presents a novel criterion for effective islanding. Section III presents the problem formulation for grid-connected and islanded operation problems. Section IV presents illustrative examples to show the proposed model applied to a microgrid. Discussion on the features of the proposed model and concluding remarks are provided in Sections V and VI, respectively.

II. MICROGRID OPTIMAL SCHEDULING MODEL OUTLINE

A. Microgrid Components

The microgrid components to be modeled in the optimal scheduling problem include loads, local generating units and energy storage systems. Microgrid loads are categorized into two types of fixed and adjustable. Fixed loads cannot be altered and must be satisfied under normal operation conditions. Adjustable loads, however, are responsive to price variations and controlling signals from the microgrid master controller. Adjustable loads could be curtailed (i.e., curtable loads) or deferred (i.e., shiftable loads) in response to economic incentives or islanding requirements. Generating units in a microgrid are either dispatchable or non-dispatchable. Dispatchable units can be controlled by the microgrid master controller and are subject to technical constraints, depending on the unit type, such as capacity limits, ramping limits, minimum on/off time limits, and fuel and emission limits. Non-dispatchable units, on the contrary, cannot be controlled by the microgrid master controller since the input source is uncontrollable. Non-dispatchable units are mainly renewable resources which produce a variable, i.e., volatile and intermittent, output power. The intermittency indicates that the generation is not always available and the volatility indicates that the generation is fluctuating in different time scales. These characteristics negatively impact the non-dispatchable unit generation and increase the forecast error, therefore these units are commonly reinforced with energy storage systems. The primary application of energy storage systems is to coordinate with generation resources to guarantee the microgrid generation adequacy. They can also be used for load shifting, where the stored energy at times of low prices is generated back to the microgrid when the market price is high. This action is analogous to shifting the load from high price hours to low price hours. The energy storage system also plays a major role in microgrid islanding applications.

B. Microgrid Optimal Scheduling Model

Fig. 1 depicts the flowchart of the proposed microgrid optimal scheduling model. The problem is decomposed into a grid-connected operation master problem and an islanded operation subproblem. The master problem determines the optimal commitment and dispatch of available dispatchable units, charging and discharging schedules of energy storage systems, schedule of adjustable loads, and the power transfer with the main grid. The optimal schedule is used in the subproblem to examine the microgrid generation adequacy and confirm an uninterrupted supply of loads for a variety of islanding scenarios. If the islanding is not feasible, i.e., microgrid does not have sufficient online capacity to supply the local load, a Benders cut, i.e., Cut 1, based on the unit commitments and energy storage system schedules is generated and sent back to the master problem for revising the current solution. The Benders cut indicates that power mismatches in the subproblem can be mitigated by readjusting the unit commitments and energy storage system schedules in the master problem. The revised solution will be examined in the next iteration of the subproblem for islanding. The iterative process continues until all islanding scenarios are feasible. It is possible, however, in some scenarios that change in unit commitments and energy storage systems schedules does not provide required online capacity to guarantee a feasible islanding. In this situation a secondary Benders cut, i.e., Cut 2, is generated based on adjustable loads schedules. This cut would revise the adjustable loads’ specified operating time interval to shift the load and accordingly enable the islanding. The inconvenience realized by consumers as a result of this change is penalized in the objective. This Benders cut indicates that power mismatch in the subproblem can be mitigated by readjusting load schedules in addition to unit commitments and energy storage system schedules in the master problem. The final solution is obtained when all islanding scenarios are guaranteed feasible. Note that Cuts 1 and 2 are represented in the form of inequality constraints which provide a lower estimate of the total mismatch in the subproblem as a function of scheduling variables in the master
problem [29].

Day-ahead schedules are calculated for the master problem and the subproblem, i.e., a 24-h scheduling horizon is considered. Any other scheduling horizon can be selected based on the master controller’s discretion without any change in the proposed model and formulation. Selection of a 24-h scheduling horizon, however, would enable microgrid master controller to benefit from day-ahead market price forecasts provided by the utility company and also keep track of energy storage system daily charging/discharging cycles. The dispatchable units’ commitments and energy storage systems charging/discharging schedules will be determined in the master problem and remain unchanged in the subproblem. The microgrid fixed load and generation of non-dispatchable units are forecasted with an acceptable accuracy. The market price at the point of common coupling, i.e., the price at which microgrid purchases the main grid power and sells excess power to the main grid, is also forecasted. It is assumed that microgrid components are highly reliable and are not subject to outage during the scheduling horizon.

C. T-τ Islanding Criterion

The microgrid must be able to switch to islanded mode at any given time in response to disturbances in the main grid. The microgrid would be resynchronized with the main grid once the disturbance is removed. The microgrid master controller, however, is not aware of the disturbance time and duration. Therefore, microgrid resources are to be scheduled in a way that local loads are supplied with no interruption using only local resources, i.e., an islanded operation, for an unknown time extent.

To characterize the microgrid capability in responding to time-varying islanding requirements a T-τ islanding criterion is proposed. T denotes the number of hours in the scheduling horizon and τ represents the number of consecutive hours that the microgrid can operate at the islanded mode. As an example, a T-2 islanding criterion requires that the microgrid be able to operate in the islanded mode for any two-hour period once it is switched from grid-connected to the islanded mode. In the two successive islanding hours the microgrid load is fully supplied from local resources since the power cannot be transferred from the main grid. This criterion represents a novel approach in ensuring microgrid resiliency and online generation adequacy in multi-hour islanding operation.

In addition to uncertainty in the microgrid islanding time and duration, forecast errors associated with the market price, the non-dispatchable unit generation, and loads, add additional uncertainty to the microgrid optimal scheduling problem. The impact of these forecast errors on microgrid optimal scheduling results is studied in Section IV. A robust microgrid optimal scheduling model to effectively capture these uncertainties will be investigated in a future work.

III. MICROGRID OPTIMAL SCHEDULING FORMULATION

A. Grid-Connected Operation

The objective of the grid-connected operation master problem is to minimize the microgrid total operation cost as follows:

\[
\text{Min} \sum_{t} \sum_{i \in G} \{F_i(P_{it})I_{it} + SU_i + SD_i\} + \sum_{t} \rho_t P_{M,t}
\]  

(1)

The first term in the objective is the operation cost of microgrid dispatchable units, which includes generation, startup and shut down costs over the entire scheduling horizon. The generation cost is commonly represented by a quadratic function, however, it could be simply approximated by a piecewise linear model. The second term is the cost of power transfer from the main grid based on the market price at PCC. When the microgrid excess power is sold back to the main grid, \(P_{M,t}\) would be negative, thus this term would represent a benefit, rather than a cost, for the microgrid. The objective is subject to generating unit, energy storage system, and load constraints, as follows:

\[
\sum_{i} P_{it} + P_{M,t} = \sum_{d} D_{dt} \quad \forall t
\]  

(2)

\[
P_{i}^{\text{max}} \leq P_{it} \leq P_{i}^{\text{max}} \quad \forall i \in G, \forall t
\]  

(3)

\[
I_{it}^{\text{min}} \leq I_{it} \leq I_{it}^{\text{max}} \quad \forall i \in G, \forall t
\]  

(4)

\[
P_{i} - P_{i(t-1)} \leq \Delta R_i \quad \forall i \in G, \forall t
\]  

(5)

\[
I_{t}^\text{on} \geq U_T(I_{it} - I_{i(t-1)}) \quad \forall i \in G, \forall t
\]  

(6)

\[
I_{t}^\text{off} \geq D_T(I_{it} - I_{i(t-1)}) \quad \forall i \in G, \forall t
\]  

(7)

\[
P_{it} \leq P_{i}^{\text{dch, min}} u_{it} - P_{i}^{\text{dch, min}} v_{it} \quad \forall i \in S, \forall t
\]  

(8)

\[
P_{it} \geq P_{i}^{\text{dch, max}} u_{it} - P_{i}^{\text{dch, max}} v_{it} \quad \forall i \in S, \forall t
\]  

(9)

\[
u_{it} \leq 1 \quad \forall i \in S, \forall t
\]  

(10)

\[
C_{it} = C_{i(t-1)} - P_{it} \quad \forall i \in S, \forall t
\]  

(11)

\[
0 \leq C_{it} \leq C_{i}^{\text{max}} \quad \forall i \in S, \forall t
\]  

(12)

\[
T_{i}^{\text{ch}} \geq M_C(u_{it} - u_{i(t-1)}) \quad \forall i \in S, \forall t
\]  

(13)

\[
T_{i}^{\text{dch}} \geq M_D(v_{it} - v_{i(t-1)}) \quad \forall i \in S, \forall t
\]  

(14)

\[
P_{dt}^{\text{min}} z_{dt} \leq D_{dt} \leq D_{dt}^{\text{max}} z_{dt} \quad \forall d \in D, \forall t
\]  

(15)

\[
\sum_{t \in \{a, b, c\}} D_{dt} = E_{d}
\]  

(16)

\[
T_{d}^{\text{on}} \geq M_U(z_{dt} - z_{d(t-1)}) \quad \forall i \in D, \forall t
\]  

(17)

The power balance equation (2) ensures that the sum of power generated by DERs (i.e., dispatchable and non-dispatchable units and energy storage systems) and the power from the main grid matches the hourly load. The forecasted generation of non-dispatchable units is used in (2), where it can be treated as a negative load. The power of energy storage systems can be positive (discharging), negative (charging) or zero (idle). The main grid power can be positive (import), negative (export) or zero. The power transfer with the main grid is limited by the flow limits of the line connecting microgrid to the main grid (3). The dispatchable unit generation is subject to minimum and maximum generation capacity limits (4), ramp up and ramp down rate limits (5)-(6) and minimum up and down time limits (7)-(8). The unit commitment state, \(I_{it}\), is one when unit is committed and is
zero otherwise. A dispatchable unit can further be subject to fuel and emission limits based on the unit type.

The energy storage system power is subject to charging and discharging minimum and maximum limits depending on its mode (9)-(10). When charging, the charging state \( v_{it} \) is one and discharging state \( u_{it} \) is zero; hence minimum and maximum charging limits are imposed. Similarly when discharging, the discharging state \( u_{it} \) is one and charging state \( v_{it} \) is zero, hence minimum and maximum discharging limits are imposed. Since the energy storage system charging power is considered as negative the associated limits are denoted with a minus sign. Superscripts \( ch \) and \( dch \) are used for charging and discharging modes, respectively. Only one of the charging or discharging modes at every hour is possible (11).

Energy storage system state of charge (SOC) is calculated based on the amount of charged/discharged power (12) and restricted with capacity limits (13). The SOC at \( t=1 \) is calculated based on SOC at the last hour of the previous scheduling horizon. It is also assumed that energy storage systems maintain similar SOC at the beginning and end of the scheduling horizon. Energy storage systems are subject to minimum charging and discharging time limits, respectively (14) and (15), which are the minimum number of consecutive hours that energy storage systems should maintain charging/discharging once the operational mode is changed.

Adjustable loads are subject to minimum and maximum rated powers (16). When load is consuming power, the associated scheduling state \( z_{dt} \) would be one; it is zero otherwise. Each load consumes the required energy to complete an operating cycle in time intervals specified by consumers (17). \( \alpha_d \) and \( \beta_d \) respectively represent the start and end operating times of an adjustable load. Certain loads may be subject to minimum operating time which is the number of consecutive hours that a load should consume power once it is switched on (18).

### B. Islanded Operation

The objective of the islanded operation subproblem for an islanding scenario \( s \) is to minimize the power mismatches as in (19).

\[
\text{Min } w_s = \sum_t (SL_{1,ts} + SL_{2,ts})
\]

\[
\sum_i P_{its} + P_{M,ts} + SL_{1,s} - SL_{2,s} = \sum_d D_{dts} \quad \forall t
\]

\[
I_{its} = \hat{l}_{it} \quad \lambda_{its} \quad \forall i \in G, \forall t
\]

\[
u_{its} = \hat{u}_{it} \quad \mu_{its} \quad \forall i \in S, \forall t
\]

\[
\nu_{its} = \hat{v}_{it} \quad \mu_{its} \quad \forall i \in S, \forall t
\]

\[
z_{dts} = \hat{z}_{dt} \quad \pi_{dts} \quad \forall d \in D, \forall t
\]

\[-P_{M,ts}^{\text{max}} U_{is} \leq P_{M,ts} \leq P_{M,ts}^{\text{max}} U_{is} \quad \forall t
\]

\[
P_{i,ts}^{\text{min}} I_{its} \leq P_{its} \leq P_{i,ts}^{\text{max}} I_{its} \quad \forall i \in G, \forall t
\]

\[
P_{its} - P_{(t-1)s} \leq UR_{i} \quad \forall i \in G, \forall t
\]

\[
P_{(t-1)s} - P_{its} \leq DR_{i} \quad \forall i \in G, \forall t
\]

\[
P_{its} \leq P_{it}^{dch,\text{max}} u_{its} - P_{it}^{ch,\text{min}} v_{its} \quad \forall i \in S, \forall t
\]

\[
P_{its} \geq P_{it}^{dch,\text{min}} u_{its} - P_{it}^{ch,\text{max}} v_{its} \quad \forall i \in S, \forall t
\]

\[
C_{its} = C_{(t-1)s} - P_{its} \quad \forall i \in S, \forall t
\]

\[
0 \leq C_{its} \leq C_{i,\text{max}} \quad \forall i \in S, \forall t
\]

\[
P_{d,ts}^{\text{min}} z_{dts} \leq D_{dts} \leq P_{d,ts}^{\text{max}} z_{dts} \quad \forall d \in D, \forall t
\]

\[
\sum D_{dts} = E_d \quad \forall d \in D
\]

Power balance equation (20) encompasses slack variables \( SL_1 \) and \( SL_2 \), which act as virtual generation and virtual load, respectively. Nonzero values for these variables denote a power mismatch in the microgrid. Unit commitments, energy storage charging/discharging schedules, and load schedules are obtained from the grid-connected operation master problem. These given variables are replaced with local variables for each scenario to obtain associated dual variables (21)-(24). Dual variables are later used in this section to generate islanding cuts.

Main grid power transfer constraint is revised by including a binary outage state, i.e., \( U_{it} \). When the outage state is set to zero, the main grid power will be zero and therefore, the microgrid is imposed to operate in the islanded mode. Islanding scenarios are generated using the outage state. In each scenario the outage state will obtain 0-1 values based on the islanding duration and will be considered in the islanded operation subproblem as an input. The islanded operation subproblem is further subject to dispatchable unit generation and ramp rate limits (26)-(28), energy storage system power and capacity limits (29)-(32), and adjustable load power and energy limits (33)-(34).

A zero mismatch for the islanded operation subproblem ensures that the microgrid has sufficient committed generation and energy storage to independently supply the local load; hence it could switch to the islanded mode without interruption in the load supply. When the objective is not zero, however, islanding Cut 1 (35) is generated and added to the next iteration of the grid-connected operation master problem to revise the current microgrid schedule.

\[
\nu_{its} + \sum_{i \in G} \lambda_{its} (I_{it} - I_{its}) + \sum_{i \in S} \mu_{ch,its} (u_{it} - u_{its}) + \sum_{i \in S} \mu_{dch,its} (v_{it} - v_{its}) \leq 0
\]

where \( \lambda_{its} \), \( \mu_{ch,its} \) and \( \mu_{dch,its} \) are dual variables of (21), (22) and (23), respectively. The islanding Cut 1 indicates that islanding mismatches can be mitigated by readjusting the microgrid schedule in the grid-connected operation master problem. Dual variables in the islanding cut are the incremental reduction in the objective function of the islanded operation subproblem. This cut results in a change in unit commitments and energy storage system schedules based on islanding considerations. The iterative process continues until power mismatches in all islanding scenarios reach zero. However, it is probable that after a certain number of iterations the islanding is not guaranteed, i.e., by revising unit commitments and energy storage system schedules a zero mismatch in all islanding scenarios is not obtained. To resolve this issue the schedule of adjustable loads would be revised using the
following cut, i.e., Cut 2:

\[ \hat{w}_s + \sum_{i \in G} \lambda_{its} (I_{r} - I_{its}) + \sum_{i \in S} \mu_{its} (u_{r} - u_{its}) + \sum_{i \in D} \mu_{ch\,i} (v_{r} - v_{its}) + \sum_{i \in D} \nu_{d\,i} (z_{d\,r} - z_{d\,its}) \leq 0 \]  

(36)

where \( \nu_{d\,i} \) is the dual variable of (24). Cut 2 enables a simultaneous change in unit commitments, energy storage system schedules, and adjustable loads schedules to guarantee a feasible islanding. To change the adjustable load schedule its specified start and end operating times are revised, in which the new operating time interval is represented by \([\alpha_d^{\text{new}}, \beta_d^{\text{new}}]\). The inconvenience for consumers due to the change in operating time interval is modeled with a penalty term (37) and added to the objective (1).

\[ \sum_{d \in D} K_d \Delta_d \]  

(37)

Additional constraints (38)-(40) are added to the grid-connected operation master problem to reflect this change. Equation (38) measures the total deviation in the operating time interval from original specified values, and (39)-(40) ensure that the new time interval spans a wider time range than the original one.

\[ \Delta_d = (\beta_d^{\text{new}} - \alpha_d^{\text{new}}) - (\beta_d - \alpha_d) \quad \forall i \in D \]  

(38)

\[ \beta_d^{\text{new}} \geq \beta_d \quad \forall i \in D \]  

(39)

\[ \alpha_d^{\text{new}} \leq \alpha_d \quad \forall i \in D \]  

(40)

The inconvenience is penalized with a constant penalty factor \( K_d \). This penalty factor could be used to prioritize the loads with regards to sensitivity in operating within the specified time intervals. A higher value for \( K_d \) represents a less flexible load in terms of operating time which gains a lower priority for time interval adjustment. The value for \( K_d \) should be selected reasonably higher than the generation cost of units and the market price, therefore the grid-connected operation master problem would consider the change in load operating time intervals as a last resort.

A period of one hour is considered for modeling the master problem and subproblems. Accordingly, islanding duration is considered as an integer multiple of one hour. Shorter time periods, however, could be considered without significant change in the proposed model. The selection of a proper time period for scheduling represents a tradeoff between the solution accuracy and the computation time. Shorter time periods would embrace more data and provide more accurate solutions while increase computation requirements.

IV. NUMERICAL SIMULATIONS

A microgrid with four dispatchable units, two non-dispatchable units, one energy storage system, and five adjustable loads is used to analyze the proposed microgrid optimal scheduling model. The problem is implemented on a 2.4-GHz personal computer using CPLEX 11.0 [30]. The characteristics of units, energy storage system, and adjustable loads are given in Tables I, II and III, respectively. The forecasted values for microgrid hourly fixed load, non-dispatchable units’ generation and market price over the 24-h horizon are given in Tables IV, V and VI, respectively.

<table>
<thead>
<tr>
<th>TABLE I Characteristics of Generating Units (D: Dispatchable, ND: Non-Dispatchable)</th>
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<tr>
<td>Unit</td>
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<th>TABLE III Characteristics of Adjustable Loads (S: Shiftable, C: Curtailable)</th>
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<td>Load</td>
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<th>TABLE IV Microgrid Hourly Fixed Load</th>
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<th>TABLE V Generation of Non-Dispatchable Units</th>
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The following cases are studied:

**Case 0:** Grid-connected microgrid optimal scheduling

**Case 1:** Optimal scheduling with T-1 islanding criterion
Case 2: Optimal scheduling with T-2 islanding criterion
Case 3: Sensitivity with regards to market price forecast errors
Case 4: Sensitivity with regards to the problem size

Case 0: The grid-connected microgrid optimal scheduling is studied for a 24-hour horizon. The DER schedule, including dispatchable unit commitment states and the energy storage system schedule, is shown in Table VII. The commitment state is 1 when the unit is on and is zero otherwise. The economic unit 1 is committed at the entire scheduling horizon as it offers a low cost power. Units 2-4 are committed and dispatched at the maximum capacity when the market price exceeds cost coefficient of these units. The energy storage system is charged at low price hours 1-6 and discharged at high price hours 16-20, shifting a total load of 10 MWh from peak hours to off-peak hours. Adjustable loads are scheduled to minimize the consumption cost and adopt to start and end times provided by the consumers as shown in Table VIII. The microgrid grid-connected operation cost is $11,183. The result indicates that the microgrid would decide on the supply source only based on economic considerations. A unit is committed only when its cost coefficient is lower than the market price. It would accordingly generate its maximum power to sell the excess power to the main grid and increase microgrid savings (i.e., to further reduce the total operation cost.) The microgrid would also discharge the energy storage system at peak hours, when the market price is at its highest, for the same economic reasons.

Case 1: The microgrid optimal scheduling is studied considering a T-1 islanding criterion. 24 scenarios are considered, each including a one hour islanding. The T-1 islanding is imposed as a robust requirement, i.e., all single hour islanding scenarios must be satisfied without causing load curtailment. In the islanded mode the power transfer from the main grid is zero, therefore sufficient capacity is committed in the grid-connected mode to enable a quick switching to the islanded mode without interruption in load supply. The optimal and feasible solution is obtained in three iterations with an execution time of 6s. The master problem solution in iteration 1 is similar to the solution in Case 0 (as shown in Tables VII and VIII.) However, this schedule results in a total mismatch of 75.63 MWh in islanding scenarios. Cut 1 is generated based on the mismatch in each islanding scenario for revising the obtained dispatchable unit commitment and the energy storage system schedule in the master problem. With the revised schedule, the second iteration total mismatch is reduced to 3.22 MWh. Since the mismatch is not zero another Cut 1 is generated and sent back to the master problem to further revise the schedule. The third iteration mismatch reaches a value of zero which means that microgrid islanding criterion is satisfied and islanding is feasible in all scenarios. The feasible islanding solution is obtained using only Cut 1, so there would be no need to form Cut 2 and change the optimal schedule of adjustable loads.

Table IX shows the commitment results considering T-1 islanding where the bold values highlight changes in the solution from Case 0. Additional units are committed in each hour to ensure an uninterrupted supply of loads when the microgrid is islanded. These units are dispatched at their minimum capacity as their generation is not economical. The energy storage system is discharged at a slower rate, i.e., 7 hours compared to 5 hours in Case 0, to cooperate in the microgrid islanding when the available unit capacity cannot completely supply the local load. The adjustable load schedule is remained unchanged, since Cut 2 is not formed, and the purchased energy from the main grid is reduced by 11.13 MWh, which is due to utilization of additional local resources. The microgrid operation cost is $11,674.55. The operation cost difference between Cases 0 and 1, i.e., $491.55, is considered as the cost of islanding.

The result indicates that when considering microgrid islanding, additional units, which not necessarily offer economic merits, have to be committed and maintained online. Although the microgrid schedule is significantly changed due to additional commitments, the total increase in the microgrid operation cost is only 4.4% compared to the grid-connected operation cost in Case 0. This small cost increase provides a huge benefit as the microgrid islanding without load interruption is ensured.

Fig. 2 depicts the main grid power transfer in Cases 0 and 1. In low price hours the power is purchased from the main grid as much as possible, i.e., 10 MW, equal to the line limit. The power purchase is reduced as the market price is increased and generation of local resources becomes more economic. The sudden increase in hour 15 is due to the fact that the market price becomes lower than cost coefficient of
dispatchable unit 4 and therefore this unit is turned off and the required power is purchased from the main grid. The main grid power transfer is almost similar in Cases 0 and 1. The minor differences in power transfer from the main grid in these two cases is a result of dispatchable units generation at their minimum capacity to enable a feasible islanding. The minimum generation of these units reduces the required power to be purchased from the main grid.

The role of the energy storage system is further investigated in this case by adding a second energy storage system with a capacity of 10 MWh, minimum-maximum rated power of 0.5 MW-2.5 MW, and minimum charging/discharging time of 4 hours. Fig. 3 compares the charging/discharging schedule of these two energy storage systems. Both energy storage systems are charged at low price hours. Energy storage system 2 with a higher charging rate is charged at lower price hours and charging of energy storage system 1 is delayed by 5 hours. An overlap between charging schedules requires additional generation of dispatchable units, however, these units are not economic at these hours and are dispatched at their minimum power output. Therefore, charging of energy storage system 1 is delayed to be supplied by the main grid power. Both energy storage systems are discharged during peak hours when the market price is high. Energy storage system 1 is discharged in an extended period of time to facilitate a feasible islanding.

Case 2: The microgrid optimal scheduling is studied considering a $T$-2 islanding criterion where the microgrid should have the islanding capability for every consecutive two-hour interruption in the main grid power. Initial grid-connected schedule results in a total mismatch of 145.73 MWh and 6.45 MWh in the first two iterations of the subproblem. The generated cut based on unit commitments and the energy storage system schedule, i.e., Cut 1, does not further reduce the mismatch, hence cannot ensure a feasible islanding at hours 17 and 18. When iteration has reached its maximum limit, here 10, the subproblem generates Cut 2, which includes the schedule of adjustable loads, and sends it to the master problem. Cut 2 is formed to reduce the mismatch, i.e., 6.45 MWh, by revising the schedule of adjustable loads in addition to dispatchable units and energy storage system. A penalty factor is added to the master problem to minimize the change in the operating time interval of adjustable loads. The penalty cost is assumed to be $100 for every hour deviation from the specified start and end times. The microgrid operation cost is reduced to $11,657.07, however an inconvenience cost of $40 (=100×0.4) is added to the total microgrid operation cost. 0.4 MW of load 2 is scheduled in hour 14 which is outside specified time interval by the consumer. Dispatchable unit and energy storage system schedules are remained unchanged compared to Case 1 (see Table IX.) The solution is obtained in 24s. The obtained result in this case illustrates that to enable the microgrid islanding not only the unit commitment and storage schedules, but in some cases adjustable loads schedules should be revised. If the microgrid cannot change the operating time interval of adjustable loads, it would have to inevitably curtail the load when in islanded mode to match the reduced load with available generation. This action would be more undesirable for consumers than revising the load operating time intervals.

Fig. 2 Main grid power transfer in Cases 0 and 1

Fig. 3 Charging/discharging schedule of energy storage systems

Fig. 4 Total operation cost as a function of number of islanding hours

The microgrid optimal scheduling problem is further solved for a variety of islanding criteria, from $T$-1 to $T$-12. Fig. 4 shows the results and illustrates that a larger number of islanding hours would increase the microgrid operation cost. This increase is a direct result of inconvenience recognized by consumers which is added to the objective as a cost. It is possible that a revised load schedule provides a feasible solution for a variety of islanding criteria, as it happened at $r=2$ to $r=5$, and also $r=8$ to $r=10$. Furthermore, the cost difference among islanding hours is very small compared to the grid-connected operation (shown at point zero in the Fig. 4.) It demonstrates that the major cost of islanding occurs at $T$-1 islanding. Additional islanding hours, however, could be performed at a small expense.

Case 3: A sensitivity analysis is performed to study the impact of forecast errors on microgrid optimal scheduling
solutions. 1000 scenarios are generated to simulate market price forecast errors based on a uniform random error of ±30% of the hourly forecasted price in Table VI. The microgrid optimal scheduling with $T$-1 islanding is performed for all scenarios. The microgrid operation cost of 1000 scenarios fall within small lower and upper bounds of [$11433.89, 11759.85$] which corresponds to [−2.06%, 0.73%] deviation from the solution in Case 1. This study shows that even with large forecast errors, acceptable solutions can be obtained using the proposed model. Similarly, 1000 scenarios are considered for load forecast error of ±10% of the hourly forecasted load. Obtained solutions deviate from the solution in Case 1 within [−3.95%, 2.96%]. Although the load forecast error is much lower than the price forecast error, deviation from the solution in Case 1 is much higher. This result suggests that the microgrid operation cost is highly sensitive to load forecast errors as small errors may translate into huge changes in the operation cost. It is worth mentioning that in the proposed method fixed and adjustable loads are modeled separately. The main source of error in load forecasts is the unpredictable schedules of adjustable loads that depend on market price and consumer preferences. The fixed load, on the other hand, can be forecasted with an acceptable level of accuracy in the short-term operation of the microgrid.

Performing a similar study for a ±30% forecast error in non-dispatchable generation, a deviation of [−1.75%, 0.96%] from solution in Case 1 is obtained. The low impact of non-dispatchable generation forecasts on the microgrid optimal operation is due to the fact that non-dispatchable generation represents a small portion of the total generation in the microgrid, which is less than 14% of the total installed capacity in the studied microgrid. Therefore, even a large change in generation of these resources would not change the results significantly. The small ratio of non-dispatchable units capacity compared to dispatchable units capacity in a microgrid is due to the fact that microgrid master controller should rely on dispatchable units for a feasible islanding in case the forecasted non-dispatchable generation is not materialized. Furthermore, non-dispatchable units offer a variable generation, i.e., the power output is not always available and may reach a zero generation for several hours during the scheduling horizon. Thus, the energy produced by these resources could be much lower than the generation of a dispatchable unit with the same size.

**Case 4:** To demonstrate the effectiveness of the proposed model in solving the microgrid optimal scheduling problem in a reasonable amount of time, the problem is solved for a variety number of adjustable loads. The number of adjustable loads is changed from 10 to 100 instead of considering only five aggregated adjustable loads as in previous cases. The microgrid optimal scheduling problem with $T$-$1$ islanding criterion is solved using both integrated and decomposed models. Fig. 5 compares the computation time in these two models. Using the proposed model, when the number of loads is increased the computation time increases almost linearly. The computation time for 100 adjustable loads is about 10 times the computation time when 10 adjustable loads are considered. Using the integrated model, by increasing the number of adjustable loads the computation time increases exponentially, where for 100 adjustable loads the computation time is larger than 100 minutes.

The proposed model decomposes the problem into a master problem and a subproblem. All binary variables associated with the dispatchable units, energy storage systems, and adjustable loads are determined in the master problem while the subproblem deals with linear variables and examines linear constraints. Furthermore, the islanding scenarios can be solved separately in the subproblem as there is no coupling constraint among islanding scenarios. Therefore, instead of solving a large-scale problem several smaller problems are solved in an iterative manner, which would significantly reduce computation time.

![Fig. 5 Comparison of computation time in integrated and decomposed models](image)

V. DISCUSSIONS

Microgrids improve the power system economics by utilizing a variety of local generation resources, energy storage systems, and adjustable loads along with energy purchase from the main grid, and increase reliability of local loads by ensuring an interrupted supply of loads when the main grid power is not available. Specific features of the proposed microgrid optimal scheduling model with multi-period islanding constraints are listed as follows:

- **Least cost operation:** The proposed model determines the optimal schedule of dispatchable generating units, energy storage systems, and adjustable loads, along with the main grid power transfer to minimize the cost of supplying local loads.

- **Seamless islanding:** The microgrid optimal scheduling is reinforced with islanded operation constraints to provide sufficient capacity for a smooth and uninterrupted supply of loads when switching to an islanded mode.

- **Islanding criterion:** The proposed novel $T$-$\tau$ islanding criterion guarantees an effective islanding for an extended period of time as the time and duration of the main grid disturbance is not known to the microgrid master controller.

- **Consumer convenience:** The consumer decisions in scheduling adjustable loads are not changed unless it is required to obtain a feasible islanding solution. The changes, however, are penalized to reduce the
inconvenience for consumers and reflect the load schedule outside specified operating time intervals.

- Model scalability and flexibility: The proposed model is comprehensive in modeling the practical constraints of microgrid components. Moreover, the proposed mixed integer programming model puts no limits on the number of components to be considered in the microgrid.

- Computational efficiency: In order to reduce the computation burdens and obtain the solution in a short amount of time, the islanding scenarios are examined as a subproblem and coupled with the grid-connected operation via islanding cuts. The decomposition reduces the size of the original problem and increases the solution speed.

VI. CONCLUSIONS

An efficient model for microgrid optimal scheduling considering multi-period islanding constraints was proposed. A novel islanding criterion was proposed for ensuring the generation adequacy of the microgrid in the islanded mode operation when the unpredicted disconnection from the main grid lasts more than one hour. The proposed criterion reflects the uncertainty in the duration of the main grid disturbance. A Benders decomposition method was employed to decouple the grid-connected operation and islanded operation problems. Islanding cuts were further utilized to couple these two problems. Mixed integer programming was used to model microgrid components, which included loads, generating units, and energy storage systems. The proposed model was analyzed through numerical simulations, where it was shown that the islanding criterion would provide significant reliability benefits while slightly increasing the microgrid total operation cost.

REFERENCES