Optimal Hourly Scheduling of Community-Aggregated Electricity Consumption

Amin Khodaei*, Mohammad Shahidehpour**, and Jaeseok Choi †

Abstract – This paper presents the optimal scheduling of hourly consumption in a residential community (community, neighborhood, etc.) based on real-time electricity price. The residential community encompasses individual residential loads, communal (shared) loads, and local generation. Community-aggregated loads, which include residential and communal loads, are modeled as fixed, adjustable, shiftable, and storage loads. The objective of the optimal load scheduling problem is to minimize the community-aggregated electricity payment considering the convenience of individual residents and hourly community load characteristics. Limitations are included on the hourly utility load (defined as community-aggregated load minus the local generation) that is imported from the utility grid. Lagrangian relaxation (LR) is applied to decouple the utility constraint and provide tractable subproblems. The decomposed subproblems are formulated as mixed-integer programming (MIP) problems. The proposed model would be used by community master controllers to optimize the utility load schedule and minimize the community-aggregated electricity payment. Illustrative optimal load scheduling examples of a single resident as well as an aggregated community including 200 residents are presented to show the efficiency of the proposed method based on real-time electricity price.

Keywords: Residential community, Hourly community-aggregated load scheduling, Real-time electricity price, Lagrangian relaxation, Mixed integer program

Nomenclature

Indices:

- \( n \) Index for adjustable load
- \( a \)
- \( b \) Index for storage system
- \( c \)
- \( f \) Index for fixed load
- \( g \)
- \( l \) Superscript for storage discharging
- \( M \)
- \( n \)
- \( r \) Index for resident
- \( R \)
- \( s \) Index for shiftable load
- \( t \)

Parameters:

- \( C_b \) Capacity of storage system b
- \( E_t \) Utility load limit at hour t
- \( NC \) Number of communal loads
- \( NR \) Number of residents
- \( NT \) Number of hours
- \( P_{at}^{min} \) Minimum consumption of adjustable load a at hour t

Variables:

- \( p_{at}^{base} \) Base consumption of adjustable load a at hour t
- \( P_{s}^{rate} \) Rated power of shiftable load s
- \( P_{f} \) Consumption of fixed load f at hour t
- \( P_{g,t} \) Local generation at hour t
- \( P_{b}^{ch}, P_{b}^{dis} \) Charging and discharging rated powers of storage system b
- \( U_{s}^{t} \) Cycle duration of shiftable load s
- \( U_{b}^{t}, U_{b}^{dis} \) Charging and discharging duration of storage system b
- \( \varepsilon \) Small positive constant
- \( \rho_{t} \) Real-time electricity price at hour t
- \( p_{at}^{min} \) Price threshold of adjustable load a at hour t
- \( \omega_{c,t} \) Priority coefficient of communal load c at hour t
- \( \omega_{r,t} \) Priority coefficient of resident r load at hour t

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Residential consumers use more than one third of the total energy consumed in the United States, representing a significant potential for demand response. The transition from the conventional utility grid to smart microgrids and the enhanced utilization of adjustable and shiftable loads have extensively changed the way communities use electricity by increasing energy efficiency, enhancing conservation levels, and lowering greenhouse gas emissions, while lowering the stress level on congested transmission lines [1-2]. The use of distributed generation for socioeconomic and environmental reasons, the enhancement of power quality and reliability, and the emergence of new types of loads in distribution networks such as plug-in vehicles and storage have accelerated the need for the design and the implementation of community aggregation for demand response. However, residential applications of demand response have faced two obstacles: lack of an effective home energy management system and lack of consumer knowledge on potential impacts of hourly load scheduling strategies [3]. The smart metering infrastructure has emerged and helped streamline the first obstacle. The second obstacle, however, still persists since the majority of consumers do not have a keen knowledge of real-time load scheduling strategies. Therefore, proper scheduling and data acquisition tools would have to be introduced to advise consumers and perform load scheduling automatically with a minimum consumer intervention. These tools may be used by individual consumers to schedule their own loads or by a central controller in a community to optimally schedule and coordinate residential loads. The financial incentives offered to consumers, who would consider load scheduling strategies according to real-time electricity prices, is the most momentous driver for adjusting consumption habits.

This paper focuses on the development of an efficient optimal load scheduling model for residential communities. Our proposed model intends to provide consumers with a simple and easy-to-use hourly load scheduling tool that would maximize consumer benefits, i.e. minimize electricity payments without any consumer interactions or compromising appliance performances. Unlike traditional direct load control approaches (in which loads are curtailed by a utility in certain circumstances regardless of consumer inconvenience), consumers define their preferences in the optimal hourly load scheduling.

The application of demand response considering intertemporal load characteristics was proposed in [4]. A real-time demand response model was presented in [5] and the impact of peak pricing on residential consumers was presented in [6] based on actual utility and consumer data. The role of communication infrastructure to support consumer demand response was described in [7]. There is also an extensive study on demand response for smart buildings. In [8], “smart building” was defined and its application to reduce energy use was explored. In [9], the available energy storage systems for smart buildings, such as batteries, ice/heat storage units, and water tanks, were analyzed and compared. Furthermore, the role of energy storage devices in reducing building energy costs was investigated. Load control strategies for air conditioner and water heater were investigated in [10]. In [11] incentive-based energy consumption scheduling algorithms was proposed and an automatic residential energy consumption scheduling framework for minimizing the payment was presented. This approach would find the optimal load schedule of a single residential consumer based on time-dependent electricity prices. In [12], a three-layer system architecture was presented for load management in smart buildings, which enabled autonomous demand side load management in the smart grid. In [13], a hierarchical multi-agent control system with an intelligent optimizer was proposed to minimize the power consumption in a smart building without compromising customer comfort. In [14], a holistic approach to reducing the energy footprint of large commercial buildings was proposed and a detailed energy use breakdown within a modern building was presented. In [15], a nonintrusive appliance load monitoring (NILM) strategy for energy management systems in smart buildings was presented. Compared to purely data-driven methods, this paper introduced a prior-knowledge-based model-driven framework. In [16], a self-adapting intelligent system was used for providing building control and energy saving services. This system consists of a gateway (self-adapting intelligent gateway) and a sensor (self-adapting intelligent sensor). In [17], smart appliances were characterized as devices that are attentive to their environment. Then, an architecture was introduced that supported the transformation from sensor data to cues. Despite extensive studies on smart buildings, very little literature is dedicated to applications to communities. In [18], a simulation environment was presented for energy management within community microgrids. The objective was to optimally regulate the supply and demand within the microgrid while ensuring that individual preferences were met and the overall energy consumption was minimized. This article only focused on the demand management within the community. The authors presented in [19] a model for the optimal operation of a community-based microgrid. The model introduced microgrid controller and consumer parameters and considered prevailing restrictions for the optimal operation of a
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microgrid. Mixed-integer programming (MIP) was used to formulate the microgrid problem. In [20], a simulation-based designing method was proposed for microgrid systems with rechargeable batteries. The design method consisted of a time-marching simulation system of a microgrid, battery management algorithm to level the electric load, and the method to quantitatively evaluate microgrids based on the simulation results. The objective of [21] was to develop an optimally-sized microgrid system composed of wind turbine, diesel generator, and hydrogen-based energy storage system for a remote community. The wind turbine was the main source of energy in [21]. The goal was to minimize the cost of supplying the hourly demand, while reducing the effect of burning fossil fuels on the environment. In [22], a study was presented for an off-grid net-zero low-energy community. In [23], a framework for establishing neighborhood-based integrated energy systems, which were mainly but not solely dependent on renewable energy, was presented. In [24], the development of a solar neighborhood microgrid concept was reported for remote communities. The approach demonstrated technologies that needed to be incorporated into microgrids with cellular-enabled solar home systems. The model proposed in [25] investigated renewable energy alternatives to reduce diesel fuel usage for electricity generation in the Ontario’s remote northern communities.

In this paper, the optimal hourly load schedule of a residential community is explored. The community controller in this model would minimize the total electricity payment, while taking into account individual residents’ preferences. Residents would define their criteria for operating specific loads by considering the cycle duration and other characteristics of individual loads. The limited energy supplied to a community will link the operation of independent households. Lagrangian relaxation [26] is applied to relax the linking constraints and decompose the original problem into a set of subproblems corresponding to each resident and the communal load. The final solution is obtained after iterations among subproblems.

The rest of the paper is organized as follows. Section 2 describes the proposed load models. Section 3 presents the proposed solution methodology of a residential community. Section 4 describes the microgrid operation mode of the residential community. Section 5 presents the illustrative examples to show the proposed model applied to practical cases. Discussion on the features of the proposed model and concluding remarks are provided in Sections 6 and 7, respectively.

2. Hourly Load Model with Specific Characteristics

In this model, fixed loads (e.g., refrigerators and microwave ovens) will not be curtailed or shifted. Adjustable loads, such as lighting, heating, and air conditioning, may be partially curtailed when the hourly electricity price is high. Only the consumption level of adjustable loads would be controlled. Heating and air conditioning loads could be adjusted by resetting the desired temperature and lighting could be adjusted by dimming the lights. The consumption level of adjustable load would be reduced to its minimum value, i.e., \( I_{at}^{min} \), when the real-time electricity price is higher than the price threshold (1). The adjustment state \( I_a \) is equal to 1 when the real-time electricity price is higher than the price threshold and is zero otherwise (2). \( \varepsilon \) is a very small positive constant. Shiftable loads may be curtailed or shifted in response to price signals and in accordance with consumer plans and preferences. The operating hours of shiftable loads would be controlled as such loads may be shifted to hours with lower electricity prices. Shiftable loads, including dishwashers, dryers, pool pumps, and plug-in vehicles, consume a fixed level of power (i.e. rated power) for a fixed period of time (i.e. cycle duration). Therefore, they are defined as a load block with a constant rated power and cycle duration. The operating time window (i.e. preferred start and end hours) is set by consumers. The shiftable loads consume the corresponding rated power when turned on (3) and remain on for the duration of their cycles (4). The shiftable loads would be turned on and operated for one cycle during the operating time window (5).

\[
P_{at} = P_{base}^{at} (1 - I_{at}) + P_{min}^{at} I_{at} \quad (1)
\]
\[
\varepsilon (P_a - P_{min}^{at}) < I_{at} \leq \varepsilon (P_a - P_{min}^{at}) + 1 \quad (2)
\]
\[
P_a = P_{rated}^{at} I_{at} \quad (3)
\]
\[
[X_{s(t-1)} - U T_s] [I_{s(t-1)} - I_{at}] \geq 0 \quad (4)
\]
\[
\sum_{t=1}^{NT} I_{at} = UT_s \quad (5)
\]

The storage system acts as a load when it is charging and a generator when discharging. Discharging is considered as a negative load (6). The charging and discharging periods are represented by (7) and (8), respectively. Equation (9) prevents simultaneous charging/discharging. Equations (10) and (11) calculate and constrain the storage state of charge (SOC), respectively. Using (10) and (11), it is ensured that at time \( t \) the storage cannot generate power if it is completely depleted (i.e., \( C_{t+1} = C_{min} \)), and also cannot store more energy if it is fully charged (i.e., \( C_{t+1} = C_{max} \)). Residential load is the summation of fixed, adjustable, shiftable and storage loads (12).

\[
P_{ht} = P_{ht} I_{ht} - P_{ht}^{min} I_{ht} \quad (6)
\]
\[
[X_{h(t-1)} - UT_h] [I_{h(t-1)} - I_{ht}] \geq 0 \quad (7)
\]
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\[ X_{bt}^g - UT_{bt}^g \geq 0 \]  
\[ I_{bt}^g \leq 1 \]  
\[ C_{bt} = C_{bt-1} + P_{bt} \]  
\[ 0 \leq C_{bt} \leq e_{max} \]  
\[ P_{nt} = \sum_{a \in R} P_{at} + \sum_{a \in R} P_{nt} + \sum_{s \in R} P_{nt} + \sum_{b \in R} P_{bt} \]  

3. Community-aggregated Model for Optimal Load Scheduling

A community is equipped with a community controller that monitors, controls and coordinates the residential and communal loads based on real-time electricity prices. The historical electricity price is used to forecast the real-time electricity price for the optimal load scheduling problem. The community controller optimally schedules the hourly community-aggregated consumption for minimizing the electricity payment over the scheduling horizon. The objective function of the community controller (13) is given as

\[ \text{Min} \sum_{t=1}^{NT} \rho_t (P_{D,t} - P_{G,t}) \]  
subject to (1-12), where

\[ P_{D,t} = \sum_{r=1}^{NR} \sum_{c=1}^{NC} P_{nt} \quad (t = 1, ..., NT) \]  
\[ P_{D,t} - P_{G,t} \leq E_t \quad (t = 1, ..., NT) \]  

The community-aggregated load (14) is the aggregation of residential and communal loads. The residential load includes individual fixed, adjustable and shiftable loads, and the communal load includes shared loads among community residents such as community storage, community pool pump, etc. The community pays for the utility load, i.e. the community-aggregated load minus the local generation. If the local generation is larger than the community-aggregated load, the excess energy is exported to the utility grid which is paid for in real-time. We assume that renewable resources alone are available at the community. The positive utility load which is purchased from the utility grid is restricted by (15). This constraint would prevent a large portion of hourly loads to be scheduled at low price hours which could create more peaks at those hours. Constraint (15) would be imposed to reflect either the community’s physical load supply limit (due to limitations on distribution system capacity, transformer limits, etc.) or the community’s bidding strategy for demand response in the electricity market. Using (15), residential and communal loads are linked in the community-aggregated optimal load scheduling problem.

Fig. 1 shows the community-aggregated optimal load scheduling for managing the utility load constraint. Mathematically, the model represents a large-scale complex mixed-integer scheduling problem. We introduce a decomposition strategy in which the Lagrangian relaxation (16) is adopted to decompose the original problem into subproblems corresponding to each residential and communal load.

\[ L = \sum_{t=1}^{NT} \rho_t (P_{D,t} - P_{G,t}) + \sum_{t=1}^{NT} \lambda_t (-E_t + P_{D,t} - P_{G,t}) \]  
\[ = \sum_{t=1}^{NT} (\rho_t + \lambda_t) P_{D,t} - \sum_{t=1}^{NT} (\rho_t + \lambda_t) P_{G,t} - \sum_{t=1}^{NT} \lambda_t E_t \]  

We assume a community only offers renewable resources; hence the community generation is uncontrollable and treated as constant (hourly values are given). Therefore, the last two terms in the Lagrangian function (16) are constant which would be dropped. The community-aggregated load can be replaced by residential and communal loads using (14). Accordingly the objective function of the problem is
which includes payments for residential and communal loads based on a pseudo price. The total residential payment is summed up over NR which can be decomposed into NR subproblems. Also the optimal communal load is solved independently. Therefore, there are NR\(+\)1 subproblems. The pseudo price \(\rho_t + \lambda_t\) is used for the optimal residential and communal load scheduling. If the schedules do not satisfy the utility load limit or the problem optimality criterion is not satisfied, the optimal load scheduling problem is recalculated by updating the Lagrangian multiplier \(\lambda_t\) (18), and obtaining a new pseudo price.

\[
\lambda^{(n+1)}_t = \max \{0, \lambda_t^{(n)} + (E_t - P_{DM} - P_{G,t}) \} \quad (t = 1, \ldots, NT)
\]  

(18)

The step size \(\alpha\), which is tuned for representing the problem characteristics, will have a larger value when the utility load constraint is not satisfied. In other words, \(\lambda_t\) is adjusted downward at a slower rate than upward. The optimality criterion is based on relative duality gap and is defined as

\[
\frac{\Gamma - \Phi}{\Phi} < \Delta
\]  

(19)

where \(\Gamma\) is the primal problem (13) subject to (1-12) and (15), and \(\Phi\) is the dual problem (16) which represents the current solution of the problem and provides a lower bound to the initial solution. The Lagrangian iterations continue until a converged solution which would satisfy the utility load limit and the optimality criterion is obtained. Proper values for Lagrangian multipliers will provide a good lower bound for the initial solution and the final solution of the relaxed problem will reach the optimal solution of the original problem [27]. Therefore, Lagrangian relaxation will offer an optimal solution with an improved computation time. Further discussions on the convergence of the Lagrangian relaxation method are found in [27-29].

4. Microgrid Model for Community-aggregated Consumption

If a major outage occurs in a utility, the community may resort to an islanding mode as a microgrid for supplying its load by utilizing local generation and applying load shedding as needed. The microgrid objective would be to maximize the supply of community-aggregated load (20):

\[
\Phi = \min \sum_{t=1}^{NT} \sum_{r=1}^{NR} (\rho_t + \lambda_t) P_{rt} + \sum_{t=1}^{NT} \sum_{c=1}^{NC} (\rho_t + \lambda_t) P_{ct}
\]  

(17)

where \(\sum_{c=1}^{NC} (\rho_t + \lambda_t) P_{ct} = 0 \quad (t = 1, \ldots, NT)

(21)

The priority coefficients are used to prioritize residential and communal loads. A larger priority coefficient represents a more critical load. In (21), where the utility supply is zero, a spillage variable \(P_{SP,t}\) is added at hours when the excess microgrid generation is spilled (i.e., potentially supplied to the utility grid). It is also possible to introduce a negative priority factor for spillage to minimize the spillage in the objective function. The community-aggregated load constraints are modified accordingly to enable microgrid load curtailments, where (22-23) are added, and (1-3) are removed from the set of constraints. The Lagrangian multiplier is updated in (24) and added to the subproblems for solving the subsequent iterations if the linking constraint is not satisfied. The iterative process will continue until (21) is satisfied and the optimality criterion is met.

\[
P_{fr,t}^M \leq P_{fr,t} \quad (t = 1, \ldots, NT)
\]  

(22)

\[
P_{fr,t}^M \leq P_{fr,t}^{base} \quad (t = 1, \ldots, NT)
\]  

(23)

\[
\lambda^{(n+1)}_t = \lambda^{(n)}_t + (P_{D,t} - P_{G,t}) \alpha \quad (t = 1, \ldots, NT)
\]  

(24)

5. Numerical Simulation

A single resident and a community with 200 residents are considered for one day to demonstrate the proposed approach for solving the optimal load scheduling problem. The proposed method was implemented on a 2.4-GHz personal computer using CPLEX 11.0 [30].

5.1 Single resident

The proposed formulation is used for a single resident, representing a smart home. The fixed load data are provided in Table 1. The adjustable and shiftable load data are provided in Tables 2 and 3, respectively. Table 2 includes combined load data for light, heating and air conditioning. A minimum adjustment of 70% is considered by residents when the real-time price is higher than 5 $/kWh. Table 3 represents the characteristics of dishwasher, dryer, pool pump, plug-in vehicle, and storage

<table>
<thead>
<tr>
<th>Hour</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
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<tbody>
<tr>
<td>Fixed Load (kW)</td>
<td>2.50</td>
<td>2.30</td>
<td>1.90</td>
<td>1.70</td>
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<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
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<tr>
<td>Fixed Load (kW)</td>
<td>1.50</td>
<td>0.80</td>
<td>1.40</td>
<td>2.10</td>
<td>3.40</td>
<td>3.80</td>
<td>3.80</td>
<td>4.10</td>
</tr>
<tr>
<td>Hour</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Fixed Load (kW)</td>
<td>5.60</td>
<td>5.80</td>
<td>5.30</td>
<td>4.40</td>
<td>4.10</td>
<td>4.30</td>
<td>3.40</td>
<td>2.80</td>
</tr>
</tbody>
</table>
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system. The real-time electricity price is obtained from utility and shown in Table 4. Local generation includes a wind turbine and a solar panel. A uniform power output of 1.5 kW is considered for wind turbine at hours 1-5 and 23-24. Similarly, a uniform power output of 2 kW is considered for the solar panel at hours 11-19. The start and ending hours are assumed to be 1 and 24, respectively, for the storage

The following cases are analyzed:
• Case 1: Base case for supplying the residential load
• Case 2: Adjustable and shiftable loads are considered in Case 1
• Case 3: Local generation is added to Case 2
• Case 4: Local storage is added to Case 3
• Case 5: Hourly microgrid operation is considered

These cases are discussed as follows:

**Case 1:** In the base case, loads are scheduled by the resident. The hourly fixed and adjustable loads are supplied as listed in Tables 1 and 2, respectively. Table 3 shows the shiftable loads which are started as required by the resident and stopped when the cycle is completed. The local generation is not considered. The total electricity payment is $11.53 per day.

**Case 2:** Fig. 2 shows the optimal residential load schedule when adjustable and shiftable loads are considered. The utility load is increased at hours 1-8 when the plug-in vehicle charging is shifted to these hours with lower hourly electricity prices. The dishwasher is operated at later hours 23-24 as compared to the base case. The pool pump and dryer are started at hours 8 and 11, respectively. The adjustable loads are reduced to their minimum at hours 13-22 when the electricity price is higher than 5 ¢/kWh. A flatter (less volatile) load profile is obtained in Case 2 compared to the base case since the peak load at hour 18 is shaved and shiftable loads are operated at low price hours. The daily saving is $1.97 as the electricity payment is dropped to $9.56. A wider window of time may further reduce the electricity payment.

**Case 3:** The local generation is added to Case 2 for supplying the residential load and further reducing the utility load. The local generation, which is directly connected to the residential network, is customized to supply the local load and enhance the local reliability. In Fig. 3, the utility load in Case 3 is compared to that in Case 2. Fig. 3 illustrates that the local supply will reduce the utility load and the residential electricity payment. The utility load is lowered at hours 1-5, 11-19 and 23-24 when the load is supplied locally by wind and solar generation. The utility load in almost the entire scheduling horizon (hours 1-5 and 9-24) is lower than that in the base case. The daily residential payment is reduced to $7.99, which shows additional savings as compared to Cases 1 and 2. The solar generates more energy and provides more saving ($1.35 vs. $0.22) than wind because the solar power is

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**Table 2. Adjustable Load Data**

<table>
<thead>
<tr>
<th>Hour</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>Adjusted Load (kW)</td>
<td>3.10</td>
<td>2.80</td>
<td>2.40</td>
<td>2.20</td>
<td>2.10</td>
<td>2.20</td>
<td>2.70</td>
<td>3.10</td>
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</table>

**Table 3. Shiftable Load Data**

<table>
<thead>
<tr>
<th>Adjustable Loads</th>
<th>Dishwasher</th>
<th>Dryer</th>
<th>Pool Pump</th>
<th>Plug-in Vehicle</th>
<th>Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power (kW)</td>
<td>0.70</td>
<td>1.50</td>
<td>1.00</td>
<td>1.40</td>
<td>1.00</td>
</tr>
<tr>
<td>Cycle Duration (Hour)</td>
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<td>1</td>
<td>3</td>
<td>8</td>
<td>4</td>
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<td>Base Start time (Hour)</td>
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<td>10</td>
<td>8</td>
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<tr>
<td>Case End time (Hour)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>User-defined Start time (Hour)</td>
<td>20</td>
<td>11</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>End time (Hour)</td>
<td>24</td>
<td>17</td>
<td>13</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>

**Table 4. Real-Time Electricity Price Data**

<table>
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<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (¢/kWh)</td>
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<td>1.90</td>
<td>1.70</td>
<td>1.50</td>
<td>1.60</td>
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<td>Hour</td>
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<td>10</td>
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<td>12</td>
<td>13</td>
<td>14</td>
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<td>16</td>
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<tr>
<td>Price (¢/kWh)</td>
<td>2.20</td>
<td>4.50</td>
<td>4.80</td>
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<td>Price (¢/kWh)</td>
<td>10.20</td>
<td>11.40</td>
<td>8.40</td>
<td>8.10</td>
<td>5.60</td>
<td>5.10</td>
<td>3.20</td>
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</table>
available in the day time when the real-time electricity price is higher. Compared to Case 1, adjustable loads and local generation respectively provide 19.05 kWh and 28.50 kWh reductions in the daily residential energy consumption. The reduction in energy consumption offered by the local generation is 1.5 times larger than that of adjustable loads, but the associated cost saving provided by the local generation is only 1.05 times higher. Therefore, load reduction is a more economical option which would provide larger savings for the resident. Adjustable loads are reduced only when the real-time electricity price is high.

**Case 4:** In Case 4, all available resources (i.e., adjustable and shiftable loads, local generation, and storage) are available for the optimal residential load scheduling. The storage system would efficiently shift the hourly load from high price hours to low price hours. The total shifted loads are 18.7 kWh which are operated at low price hours. The inclusion of storage reduces the residential electricity payment to $7.67 per day. Fig. 4 compares the utility loads in Cases 3 and 4. The storage which is charged at off-peak hours 2-5 and discharged at peak hours 17-20, would shift a portion of the residential peak load to off-peak hours. A residential load of 4 kWh is shifted which is limited by the size of storage. A larger storage system with a higher load shifting capability would cost more to install, but could provide additional savings to consumers in real-time.

The proposed algorithm for the optimal hourly scheduling of the community-aggregated electricity consumption would adjust loads and utilize the storage and the local generation for satisfying residential energy requirements while reducing electricity payments. Fig. 5 compares Cases 1 and 4 and shows that the optimal hourly residential scheduling could reduce the utility load and the electricity payment. The daily residential energy consumption is 150.95 kWh in Case 4 which is lower than the 198.5 kWh given in the base case. In Fig. 5, the utility load is mostly changed which indicates that the optimal load scheduling is a viable option for reducing the electricity payment.

Table 5 summarizes the electricity payment in Cases 1-4 along with the associated cost savings. The highest saving occurs in Case 4 when considering all available resources for residential demand response. Since there is no linking constraint among loads, storage, and local generation, they could be optimized individually in this case.

**Case 5:** The objective in this case (reliability case) is to maximize the hourly residential load supply by a microgrid rather than minimizing the residential electricity payment (economic case). Residential loads are prioritized in which the fixed loads have a higher priority. The optimal residential load scheduling problem is solved in which the residential fixed load is partially supplied at hours 1-5 and 11-24, and curtailed at the remaining hours, since there is no local generation available at hours 6-10. The storage is charged at hours 11-14 to prevent spillage at hour 11 and is discharged at hours 20-23 to partially supply the fixed load at these hours. The adjustable and shiftable loads are not scheduled (are curtailed) since they have lower priorities and there is insufficient generation available to supply these loads. The total residential energy consumption is 28.5 kWh (same as the total energy generated by local resources) as compared to 198.5 kWh in Case 1. The hourly utility load in the microgrid mode is zero (i.e., microgrid will not be fed by the utility). The load shedding at this case may be reduced by the availability of additional local generation, which may have to bear installation costs.

### 5.2 Community-aggregated Load

A community consists of 200 residents, where each resident includes fixed, adjustable and shiftable loads, and residential storage system. The community includes an 800 kWh community storage system at 200 kW, a 150 kW wind turbine and a 200 kW solar panel. Random data for operating each appliance is considered. The data for this system are given in http://motor.ece.iit.edu/data/community.xls. Two cases are studied as follows.
Case 1: Optimal load scheduling without any limitations on the utility supply

Case 2: The utility load in Case 1 is limited

**Case 1:** Fig. 6 depicts the utility load in base case and the optimal load schedule. In the optimal load case, the utility load at hours 6-23 is reduced by the adjustable loads, storage systems, and local generation. The residential and community storage systems are charged at hours 1-4 and discharged at hours 16-19. The charging of storage systems increases the utility load at low price hours 1-4. The local generation reduces the utility load at hours 6-19 and 23-24, and adjustable loads reduce the utility load at hours 13-22. The shiftable loads are scheduled based on residents’ preferences. A significant drop of more than 1200 kW in the utility load at hours 16-18 is due to adjustable loads, availability of solar generation, and discharge of storage systems. Compared to the base case, the daily community energy consumption is reduced by 8.22 MWh (using adjustable loads and local generation), and shifted by 2.18 MWh to low price hours (using storage systems and shiftable loads). The community-aggregated daily electricity payment is reduced from $2,664.45 to $1,930.26, which shows a 28% reduction. Fig. 6 illustrates that by coordinating residential and community loads, a large portion of the utility load is shifted to low price hours 1-4.

**Case 2:** A 1,500 kW utility load limit is imposed to hours 1 and 2. Accordingly, the community and residential storage systems will have a different schedule as compared to Case 1. Fig. 7 depicts the optimal utility load with and without the utility load limit. The community storage is charged at hours 3-6 instead of 1-4 which reduces the utility load. The storage is discharged at hours 16-19. In addition, 75 residential storage systems are charged at hours 3-6 instead of 1-4 to reduce the utility load which was originally violated at hours 1 and 2. The community-aggregated electricity payment is slightly increased in Case 2 to $1,932.41 for satisfying the utility load limit. The solution obtained in 26sec illustrates that the utility load limit is satisfied by scheduling residential and community storage. The solution is obtained in 2 iterations with a relative duality gap of 0.001.

To provide an insight on the economics of storage and local generation, the load scheduling is optimized next for a year and the benefits of storage and local generation are calculated for a variety of sizes. This calculation is based on annual forecasts for community-aggregated loads and real-time electricity prices. We change the community storage size between 0-500 kW which results in an average saving of $4,640 per 100 kW increase in the storage size. So an annualized capital cost of less than $46.4/kW will justify the installation of additional storage system. Similarly, savings of $52.4/kW and $88.2/kW are obtained for utilizing wind and solar generation, respectively.

### 6. Discussions and Observations

Specific features of the proposed algorithm for the optimal hourly scheduling of community-aggregated electricity consumption are listed as follows:

- Residential load scheduling: Residential loads are categorized and modeled. The efficient modeling of loads is a critical step in demand response and the optimal scheduling of community-aggregated load supply.

- Community-aggregated load scheduling: The hourly residential and communal loads are coordinated by the master controller for the community cost saving while considering residential preferences and utility load limits.

- Microgrid operation: Local generation and community storage provide viable opportunities for supplying the community-aggregated loads in an islanded microgrid. In this case, loads are prioritized to maximize the local supply.

- Economic benefits: Local generation would satisfy local loads, storage would shift peak loads to off-peak hours, and adjustable and shiftable loads would be scheduled for the economic benefits to the community. Individual residents could benefit from the proposed optimal scheduling for reducing electricity payments.

- Computational efficiency: An effective decomposition approach, which is based on Lagrangian Relaxation, is employed to separate residential and communal...
problems. The final iterative solution satisfies the optimal community-aggregated load schedule and meets the utility load limit. The reasonable execution time makes the proposed method applicable to large-scale cases.

7. Conclusions

In this paper, an optimal hourly load scheduling considers the residential demand response in a large residential community. The actual operating characteristics of individual appliances and the modeling of fixed, adjustable and shiftable loads, storage, and local generation are considered. The optimal scheduling of community-aggregated loads which considers residential preferences and the utility load limit is based on real-time electricity prices. The decomposition of community-aggregated problem into residential and communal load scheduling subproblems uses Lagrangian multipliers which are updated iteratively until the violations are alleviated. Numerical simulations demonstrate the effectiveness of the proposed optimization formulation.

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References


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