On the Optimal Selection of Motors and Transmissions for Electromechanical and Robotic Systems

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***Abstract*— With regard to the important role of motors and transmissions in the performance of electromechanical and robotic systems, this paper intends to provide a solution for the problem of selection of these components for a general load case. Appropriate objectives are formulated, and by the use of them, a procedure is suggested to compare the performance of different motors for a specified task. Moreover, considering different limitations, the range for feasible transmission ratios is analytically obtained and suggestions for choosing a transmission ratio from this range, and if available, motor torque constant, are provided. As a case study, the methods are applied to the problem of actuator design for a legged robot.**

I. INTRODUCTION

Servomotors are the essential source of motion and force in numerous robotic and other electromechanical systems. Therefore, design and appropriate selection of these systems have a vital role in the desirable functionality of them. In this way, the ability of the servo to accomplish the requested task, its efficiency, and its performance are usually the three main aspects which are considered as the determining factors for selection of the motors and the accompanied transmission system. However, the weight on each of these factors and the level of complexity of modeling have varied in the literature.

Due to the commonness of the problem, there have been numerous research works and guidelines to select a motor-transmission combination for a specified task. The classically well-known method of Pasch and Seering [1], leading to choosing the gear ratio for matching the effective inertia of the motor to that of the load, is essentially the result of minimizing energy consumption of a chosen motor for a purely inertial load. At the other end of the spectrum is the approach taken by Chen and Tsai [2], [3] where they introduced and optimized for a parameter that they named “acceleration capacity” of robotic end-effectors, as an example of optimizing for agility.

A deeper study was presented in a series of articles by Van de Straete et al. [4], [5], [6]. In their first work [4], they introduced a method for determining the capability of motors, based on their speed and torque limits, to accomplish a specified task regardless of the transmission ratio. As well, for each motor they obtained a feasible transmission ratio range, based on the same limits. In their next effort [5], they extended their method by considering different types of motors as well as formulating each of the aforementioned limiting factors. Finally, in [6], by limiting the load to merely an inertial one, they designed a variable-ratio transmission to optimize energy. The works presented by Cusimano [7], [8], [9], [10], are essentially based on Van de Straete’s works in parameterizing torque and speed by square root of rotor moment of inertia. He added different features and details to the aforementioned analyses; however, the level of details considered in some aspects, in addition to making the procedure difficult to apply and use, seems not in accordance with oversimplifications in other aspects (as we will discuss throughout this paper). A very practical approach has been taken by Roos et al. [11] for gearbox selection of electric vehicles, and it basically relies on plotting the motor attributes for all combinations of available motors and gearboxes, and then deciding on the best available choice. A good balance between level of details and practicality is the work presented by Giberti et al. [12] in which they start with an analytical approach by considering the parameterized torque and speed (a la Van de Straete [4]), and then present a method for selection using a graphical representation.

In works such as [13] and [14], methods for optimization of drive-train for multiple degrees of freedom (with applications in industrial robotics) have been presented. The objective in these works are parameters such as mass and/or price which can simply be added together for different joints. Although it is a reasonable approach for a quick analysis of high-DoF systems, for lower number of joints, separate optimization of each joint can lead to more insightful selection of the actuators.

In the present work, we aim to extend the previous works to provide more thorough and more precise guidelines for selection and comparison of motors and their associated transmission systems. Accordingly, a more accurate electromechanical model is used for motors to simulate their behavior and obtain their operation limits. Furthermore, in addition to the commonly-used notion of energy minimization (which will be used with some corrections and extensions), we utilize another renowned concept, namely bandwidth, which, to our knowledge, has not been used in the study of servo selection before. These two parameters will provide a measure for transmission ratio optimizations as well as a way for comparison of different motors for the specified task and for choosing the more appropriate one. Thereafter, for the cases where rewinding of the motor is available, we provide an algorithm to include winding as an additional optimization variable. Finally, based on the methods proposed, a case study on the actuator design for a legged robot (an application in which both power and

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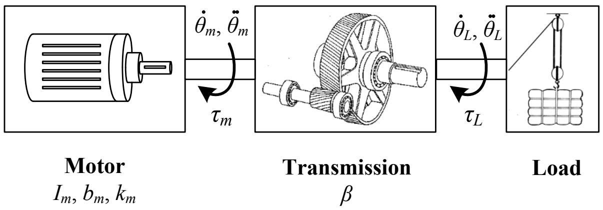

Fig. 1. Schematic of the system (The icons are for depiction purpose only).

bandwidth of the actuator are of crucial importance) will be presented and discussed.

II. System Modeling

The system under study, as shown in Fig. 1, consists of three main parts, namely: load, transmission, and motor. In this section, the approach for modeling each of them and the corresponding assumptions are presented and discussed.

A. Load

The fact that considering the load as purely inertial makes the analysis merely a kinematic one, to a great extent simplifies the problem. As a result, a lot of authors have used this assumption to deal with a less complex problem. For example, while Van de Straete et al. in their motor selection papers [4], [5] had considered a general load, in the more complicated problem of solving for a variable-ratio transmission, the load is purely inertial [6].

Throughout this work, we always consider a dynamic load. i.e. the force and the displacement are completely independent and both are set by the requirements of the problem as time trajectories τ_L and θ_L (as in Fig. 1).

B. Transmission

Transmission is the component that converts (typically reduces) the motor speed to the load speed with a characteristic ratio which can be constant or variable. Some examples of the transmission systems are gears, cable drives, and linkages. Regardless of being constant- or variable-ratio, every transmission system is accompanied by a power loss. Although there have been studies on the dependence of this efficiency on speed and other variables [15] for other purposes, since the variation is small and does not affect the results of this analysis, in the present work, the efficiency of all transmission systems are assumed to be constant. This has been the case in most of other works on this subject such as [10]. With this assumption, the power loss can be taken into account as a scaling term to τ_L on the load side and the complexities in works such as [16] (which has also assumed piecewise constant efficiencies) is avoided.

C. Motor

Brushless DC motors have several advantages over other types, including high efficiency, high lifetime, and low noise. These reasons have led to their prevalent usage in industrial and robotic applications. Therefore, the motors considered for this research are brushless DC motors. However, the methods presented are not limited to this type of motors, and the extension to other types is straightforward.

To investigate the performance of the motors, two main attributes are considered in this work. The first attribute, which was considered in the previous works, is the drawn power. Since optimizing only for energy usually leads to high gear ratios (which make the system slow and unsuited for applications such as force and impedance control), a second attribute is required to be considered. This drawback is best parameterized by bandwidth frequency. As a result, we propose to use this parameter as the second objective. In control of servo systems, bandwidth is especially important, as it is a measure of the ability of the system for fast response and fast rejection of disturbances. The closest related concept in the literature related to this topic is perhaps the acceleration capacity introduced by Chen and Tsai [2] which, being a time attribute, cannot completely capture the frequency-response characteristics of the system.

In the following subsections, these two parameters are discussed in further details.

1) Power: The total power of an electric motor can be divided into three parts: 1) p_s, the output power; 2) p_m, the power for driving the motor shaft; and 3) p_e, the power dissipated due to the electrical resistance:

$$p_{tot} = p_s + p_m + p_e \tag{1}$$

Referred to Fig. 1, for the output power we have $p_s = \tau_L \dot{\theta}_L$. As well, the power for driving the motor shaft can be obtained as: $p_m = I_m \ddot{\theta}_m \dot{\theta}_m + b_m \dot{\theta}_m^2$, wherein I_m and b_m are motor mass moment of inertia and damping, respectively. Finally, the electrical power loss is:

$$p_e = \frac{\tau_m^2}{k_m^2} \tag{2}$$

where $k_m = \frac{k_t}{\sqrt{R}}$ is the motor constant, k_t is the torque constant, and R is the winding resistance. Replacing the motor torque, τ_m, into (2), and taking $\beta = \frac{\dot{\theta}_m}{\dot{\theta}_L}$ as the gear ratio:

$$p_e = \frac{1}{k_m^2}(I_m \ddot{\theta}_m + b_m \dot{\theta}_m + \frac{\tau_L}{\beta})^2 \tag{3}$$

As motor constant is independent of motor winding (unlike torque and speed constants), parameterizing in terms of this variable makes the analysis valid for customized windings as well.

2) Bandwidth: As it can be inferred from Fig. 1, from the control point of view, the input to the system is torque and the output can be taken either velocity or position. For the present study, the velocity is considered as the output. This is particularly due to the fact that mechanical impacts (i.e. disturbances) cause discontinuities in velocity and not in position, and the ability for rapid regulation of this variable (back to the desired value) is critical.

To obtain the bandwidth, ω_b, with β as the gear ratio and input and output as chosen in the above, and with reference to Fig. 1, in Laplace domain we have:

$$\Omega_L(s) = \frac{T_m(s) - \frac{T_L(s)}{\beta}}{\beta(I_m s + b_m)} \tag{4}$$

where $\Omega_L(s) = s\Theta_L(s)$ is the output speed. Then the amplitude of the frequency response will be:

$$|\Omega_L(j\omega)| = \frac{|\frac{T_m(j\omega)}{\beta} - \frac{T_L(j\omega)}{\beta^2}|}{\sqrt{I_m^2\omega^2 + b_m^2}} \tag{5}$$

Solving for frequency:

$$\omega = \frac{\sqrt{\frac{(\frac{T_m}{\beta} - \frac{T_L}{\beta^2})^2}{\Omega_L^2} - b_m^2}}{I_m} \tag{6}$$

Using $\tau_m = \tau_{m,max}$ from motor specifications, $\tau_L = \tau_{L,max}$ from the maximum of the load torque (i.e. worst case), and ω_L as a predefined value for the amplitude of the output signal that the system is to be able to follow (typically from the maximum speed in the load side), one can obtain the maximum frequency that the system can follow from (6).

III. Constant-Ratio Transmission

Once the model and the objectives are formulated, the next step is obtaining the optimal values. In this section, the optimal selection of a constant-ratio transmission is investigated and derived.

For a given motor, the choice of a constant transmission ratio can be investigated in terms of optimization of the two entities discussed in Section II-C, i.e. energy and bandwidth. In what follows, we analytically formulate and investigate each of these objectives as well as their combination.

A. Optimization for Energy

In order to obtain the optimal ratio for the minimization of energy, from (1) we have:

$$\begin{aligned} p_{tot} = p_s + p_m + p_e &= \tau_L\dot{\theta}_L + I_m\ddot{\theta}_m\dot{\theta}_m + b_m\dot{\theta}_m^2 \\ &+ \frac{1}{k_m^2}(I_m\ddot{\theta}_m + b_m\dot{\theta}_m + \frac{\tau_L}{\beta})^2 \end{aligned} \tag{7}$$

Substituting:

$$\dot{\theta}_m = \beta\dot{\theta}_L \quad \text{and} \quad \ddot{\theta}_m = \beta\ddot{\theta}_L \tag{8}$$

and integrating, the total energy drawn from the electrical source, E_{tot}, is obtained as:

$$\begin{aligned} E_{tot} = & \int_0^T \tau_L\dot{\theta}_L \mathrm{d}t \\ & + I_m(1 + \frac{2b_m}{k_m^2})\beta^2 \int_0^T \dot{\theta}_L\ddot{\theta}_L \mathrm{d}t \\ & + b_m(1 + \frac{b_m}{k_m^2})\beta^2 \int_0^T \dot{\theta}_L^2 \mathrm{d}t \\ & + \frac{I_m^2}{k_m^2}\beta^2 \int_0^T \ddot{\theta}_L^2 dt + \frac{1}{k_m^2\beta^2}\int_0^T \tau_L^2 \mathrm{d}t \\ & + \frac{2I_m}{k_m^2}\int_0^T \tau_L\ddot{\theta}_L \mathrm{d}t + \frac{2b_m}{k_m^2}\int_0^T \tau_L\dot{\theta}_L \mathrm{d}t \end{aligned} \tag{9}$$

Noting that all of the seven integrals of (9) depend only on load characteristics and replacing them with A_1 to A_7, respectively, we obtain:

$$\begin{aligned} E_{tot} = & A_1 + \tfrac{2I_m}{k_m^2}A_6 + \tfrac{2b_m}{k_m^2}A_7 + \tfrac{A_5}{k_m^2\beta^2} \\ & + \left[I_m(1 + \tfrac{2b_m}{k_m^2})A_2 + b_m(1 + \tfrac{b_m}{k_m^2})A_3 + \tfrac{I_m^2}{k_m^2}A_4\right]\beta^2 \end{aligned} \tag{10}$$

Setting $\frac{dE_{tot}}{d\beta} = 0$ for obtaining the transmission ratio that minimizes the energy and solving the algebraic equation results in:

$$\beta_e = \sqrt[4]{\frac{A_5}{I_m(k_m^2 + 2b_m)A_2 + b_m(k_m^2 + b_m)A_3 + I_m^2 A_4}} \tag{11}$$

Equation (11) is significant in that the energy-optimal transmission ratio in the most general case in terms of load characteristics and motor parameters can be calculated from it. Other formulas provided in literature are all special cases of (11). For example, assuming that the motor damping is negligible (which is generally not right), setting $A_2 = 0$ (for example for a cyclic load), and replacing A_4 and A_5 by the corresponding Root Mean Square (RMS) values, (11) simplifies to:

$$\beta_e = \sqrt{\frac{\tau_{L,rms}}{I_m\ddot{\theta}_{L,rms}}} \tag{12}$$

which is the expression obtained in [12].

Equation (12) can be even further simplified by assuming that the load is a purely inertial one. In this case $\tau_L = I_L\ddot{\theta}_L$, where I_L is the load inertia. Substituting in (12), we obtain $\beta_e = \sqrt{\frac{I_L}{I_m}}$, which is essentially the renowned classic method of matching the load's and the motor's effective inertias for calculation of optimal gear ratio [1].

B. On Optimization for Bandwidth and its Relation with Energy

From (6) one can see that for a given motor, maximizing the bandwidth frequency is equivalent to maximizing $\frac{\tau_{m,max}}{\beta} - \frac{\tau_{L,max}}{\beta^2}$. The maximum for this expression occurs at the following transmission ratio:

$$\beta_b = \frac{2\tau_{L,max}}{\tau_{m,max}} \tag{13}$$

which, correspondingly, maximizes the actuator bandwidth.

In general, the transmission ratio calculated from (13) is different from what is obtained from (11). The primary solution for such problems is optimization of the weighted sum of the objectives, which naturally raises the question of finding "the best" weights. A second way can be considering one of the objectives (e.g. energy) as the primary objective and taking the second one (bandwidth in this case) as a constraint with a specified minimum. The third alternative that we will use is the concepts of multi-objective optimization, as will be explained in the next section.

IV. Motor Selection

Based on the parameters presented in the previous sections, in this section, we provide methods for comparison and selection of motors for a specified task. In this way, first we determine the range of transmission ratios that can handle a task within the limits of operation of a selected motor, and then show the trade-off between power and bandwidth as the transmission ratio changes.

A. Range of Transmission Ratios

To determine the limits for motor operation we consider the conventional linear characteristics typically used for the DC motors (Fig. 2). This characteristic is essentially based on the assumption of unchanging torque constant and electrical resistance in the range of operation of the motor (which are reasonable assumptions for this analysis), and the limitation to this line is basically due to constant supply voltage. Note that in all previous works on this topic this important limit has been neglected, perhaps because of the assumption of being connected to a high-voltage source. However, in many application involving interaction with humans, lower voltages ($< 50V$) are enforced. Furthermore, in many applications the power source is the limited voltage of the portable batteries. As a result, in order for the analysis to be applicable to such cases, the linear-characteristic limit must be considered.

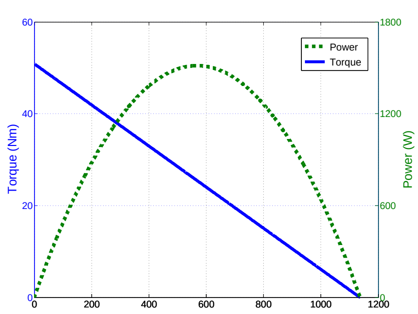

Fig. 2. A typical DC motor's characteristics

To be in the operation range, we need:

$$|\dot{\theta}_m| \leq \omega_{nl} - \frac{|\tau_m|}{k_m^2} \tag{14}$$

where ω_{nl} indicates the no-load speed of the motor. Substituting $\dot{\theta}_m$ and τ_m in terms of load parameters, and assuming that the critical conditions occur during positive motor power (i.e. when $\mathrm{sgn}(\dot{\theta}_m) = \mathrm{sgn}(\tau_m)$), (14) is simplified to:

$$\left| \beta\dot{\theta}_L + \frac{1}{k_m^2}\left[\left(I_m\ddot{\theta}_L + b_m\dot{\theta}_L\right)\beta + \frac{\tau_L}{\beta}\right]\right| \leq \omega_{nl} \tag{15}$$

In order for the inequality of (15) to have real number solutions, the following condition must hold:

$$\left[\dot{\theta}_L + \frac{1}{k_m^2}\left(I_m\ddot{\theta}_L + b_m\dot{\theta}_L\right)\right]\tau_L \leq \frac{1}{4}k_m^2\omega_{nl}^2 \tag{16}$$

It can be shown that the right hand side of (16) is the unexceedable maximum motor power, as in Fig. 2. Although the total motor power depends on transmission ratio (C.f. (1) and (3)), the significance of (16) is in providing an analytical first check for the feasibility of a given a task and a given motor regardless of the transmission ratio.

Now, assuming (16) is satisfied, solving (15) provides the range of transmission ratios:

$$\max_t\{|\beta_2|\} \leq \beta \leq \min_t\{|\beta_1|\} \tag{17}$$

where

$$\beta_{1,2} = \frac{\omega_{nl} \pm \sqrt{\omega_{nl}^2 - \frac{4\tau_L\omega_{nl}}{\tau_s}\left[\dot{\theta}_L + \frac{\omega_{nl}}{\tau_s}(I_m\ddot{\theta}_L + b_m\dot{\theta}_L)\right]}}{2\left[\dot{\theta}_L + \frac{\omega_{nl}}{\tau_s}(I_m\ddot{\theta}_L + b_m\dot{\theta}_L)\right]} \tag{18}$$

and τ_s is the stationary motor torque as in Fig. 2.

In addition to the torque-speed linear limit, another limit that must be considered is the torque limit. This limit is imposed either due to demagnetization torque of the motor, or due to the maximum current that the drive can provide. Denoting the minimum of these two values by $\tau_{m,max} := \min\{k_t i_{max}, \tau_{demag}\}$, we need to have:

$$\left|(I_m\ddot{\theta}_L + b_m\dot{\theta}_L)\beta + \frac{\tau_L}{\beta}\right| \leq \tau_{m,max} \tag{19}$$

Similar to above, for having an answer for this inequality it is required that:

$$\tau_L(I_m\ddot{\theta}_L + b_m\dot{\theta}_L) \leq \frac{1}{4}\tau_{m,max}^2 \tag{20}$$

With this condition satisfied, solving (19) results in:

$$\max_t\{|\beta_4|\} \leq \beta \leq \min_t\{|\beta_3|\} \tag{21}$$

where

$$\beta_{3,4} = \frac{\tau_{m,max} \pm \sqrt{\tau_{m,max}^2 - 4\tau_L(I_m\ddot{\theta}_L + b_m\dot{\theta}_L)}}{2(I_m\ddot{\theta}_L + b_m\dot{\theta}_L)} \quad (22)$$

Equations (17) and (21) imply another condition for existence of a transmission ratio. Denoting:

$$\beta_l = \max\left\{\max_t \{|\beta_2|\}, \max_t \{|\beta_4|\}\right\} \quad (23)$$

and:

$$\beta_u = \min\left\{\min_t \{|\beta_1|\}, \min_t \{|\beta_3|\}\right\}, \quad (24)$$

then to have a nonempty range for β we must have:

$$\beta_l \leq \beta_u \quad (25)$$

In summary, for a given task and a given motor, once (16), (20), and (25) are satisfied, then the applicable range of transmission ratio will be:

$$\beta_l \leq \beta \leq \beta_u \quad (26)$$

in which β_l and β_u are calculated from (18), (22), (23), and (24).

B. Comparing the Motors

Based on the objectives introduced in the Sections II-C (energy and bandwidth) and the obtained limits for transmission ratios, in this section, a method for comparison of the motors for a specified task is presented.

As we have two objectives, we base our method on the multi-objective optimization concepts, and specifically Pareto dominance [17].

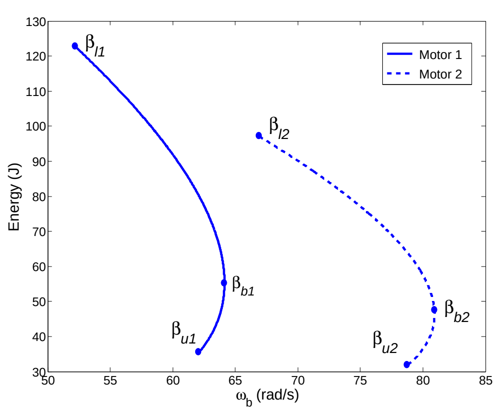

Fig. 3. Performance of two motors for a given task as the transmission ratios changes. Note that for this special cases we have $\beta_e > \beta_u$, and thus the optimal energy ratio is not a part of the applicable transmission ratios in either case, and as a result does not appear in the figure.

Fig. 3 shows the performance of two motors for a given task, as functions of transmission ratios in the feasible ranges. Since optimality is equivalent to more bandwidth and less energy, one can conclude that motor 2 has a superior performance for the given task, as it is dominant in the sense of Pareto. In other words, for each point on the motor 1 curve, there is at least one point for motor 2 with less power and more bandwidth.

Note that although in the above example we compared the whole range of the applicable transmission ratios, the significant section of the curve is the interval between the two optimal ratios for energy and bandwidth; i.e. $\max\{\beta_l, \beta_b\} \leq \beta \leq \min\{\beta_u, \beta_e\}$ (as normally $\beta_b < \beta_e$). Accordingly, as in general the curves intersect each other and as a result one motor is not completely dominant over the other, one can compare their performances in the "important" parts of the curves and choose the more appropriate motor. As well, once the motor is selected, the corresponding gear ratio can be chosen from the above range, depending on which one of the two objectives is more important. If bandwidth is the determining factor, the ratio will be closer to β_b, whereas if there is more concern about efficiency, the ratio will be chosen near β_e. We will further investigate this process in Section V.

C. Extension to the Motors with Customized Windings

Thus far we have implicitly assumed that the the winding of the motor is not changing, and as a result, its torque constant (which in turn affects the operation limits of the system, ω_{nl} and τ_s), is constant. This assumption (which has been the basis for all previous works on this subject) is appropriate for off-the-shelf selection of the motors. However, since a lot of companies provide the option of customized windings per request, and because the range of the transmission ratios depends on torque constant, in this section, we propose a method for considering winding changes in this analysis.

As before, the objectives we consider for assessing the performance of the system are energy and bandwidth. However, since in this case there are two independent variables, an analytical solution for the range of the feasible design variables cannot be obtained and thus we use a numerical approach. Since there are two objectives and two independent variables, the problem takes the standard form of a multi-objective optimization problem:

$$\begin{bmatrix} \beta^* \\ k_t^* \end{bmatrix} = \arg\min_{\beta, k_t} \left\{ \begin{bmatrix} E_{tot} \\ -\omega_b \end{bmatrix} \right\} \quad (27)$$
$$\text{s.t. } \{(15), (19)\}$$

wherein β^* and k_t^* are the optimal variables that construct the Pareto front, and the constraints of the previous section (due to voltage limit and torque limit) are applied in this case, as well. By solving optimization problem (27) using any multi-objective optimization algorithm and obtaining the Pareto optimal surface, one can choose an appropriate winding and transmission ratio for the motor, based on the application and the associated importance of each of the objectives.

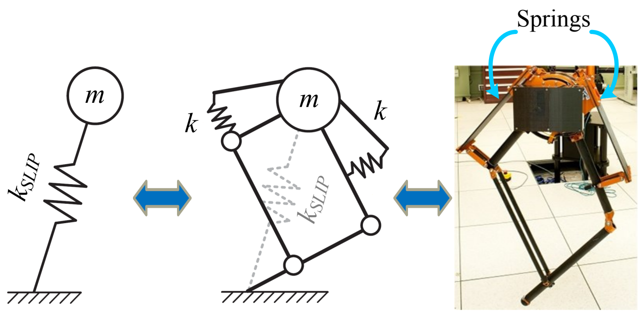

Fig. 4. ATRIAS' legs have been designed based on SLIP template.

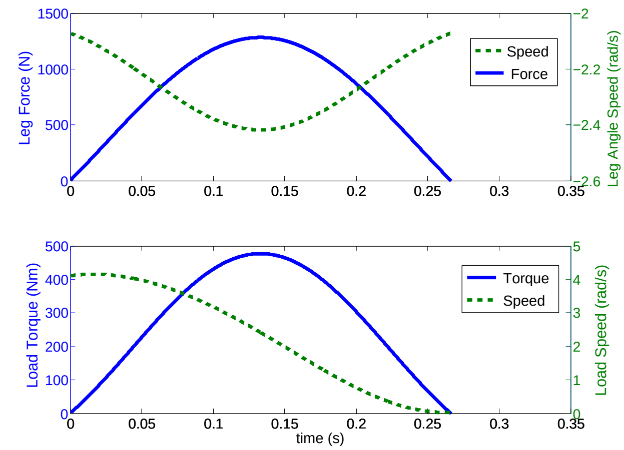

Fig. 5. SLIP running gait (top), and kinematically transferred load to the motor joint (bottom)

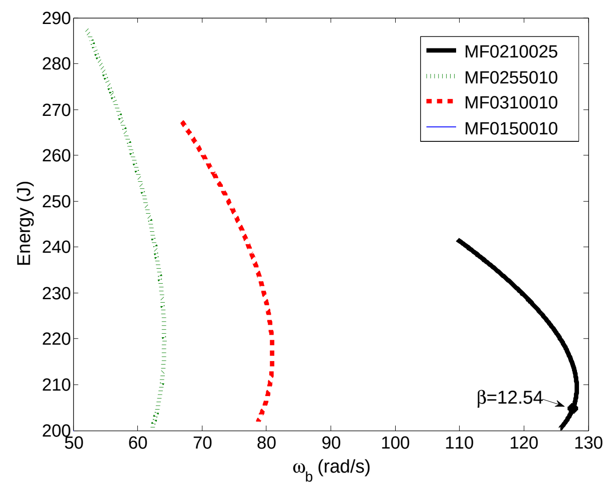

Fig. 6. Trade-off plots and Pareto dominance for the four motors considered for ATRIAS

To compare different motors, similar to the case of the unchanged winding, we use the concept of Pareto dominance. However, in this case, the analytical expression of objectives in the feasible range is replaced by the Pareto front obtained for each motor from (27). Then the concept of dominance, in the same way as for the unchanged winding case in the previous section, can be applied to select the best motor. The procedure will be detailed through the case study presented in the next section.

V. Case Study: Actuator Selection for a Legged Robot

One of the important examples of application of the analysis presented in this paper is for legged robots. Electrically actuated legged robots, both because of relying on batteries as the source for power (which necessitates power saving and also imposes limited voltage), and the requirement for fast responses to maintain their stability (because of their interaction with unpredictable environments) are extremely dependent on the performance of their servo systems.

In the present case study, we apply the methods proposed in the previous sections to ATRIAS, a bipedal robot designed and built in Dynamic Robotics Laboratory of Oregon State University. ATRIAS' leg mechanism have been designed based on the biomechanical notion of Reduced Order Models [18], [19] to realize the Spring Loaded Inverted Pendulum (SLIP) template [20] in a real robot (Fig. 4). Each of the springs of ATRIAS are actuated by a motor grounded to the body. Designing based on template enables one to take advantage of the control strategies and the gaits obtained for the SLIP system [21], [22]. Hence to select the motor and the transmission ratio, we use a SLIP-inspired running gait as the nominal load of the servo system. Based on the kinematic analysis of the leg mechanism, the desired load on the motors for following the gait is obtained. Due to symmetry, the torques are equal but in opposite direction for the two motors. Therefore, we limit our analysis to one of the motors.

Fig. 5 depicts the SLIP gait and the corresponding load on the motor. Note that since the displacement in spring direction in SLIP is the same as spring deflection of ATRIAS in the same direction, and as a result it does not affect the motors of the robot [23], only displacement in leg angle (tangential) direction has been shown. Having the load characteristics, and assuming unchangeable windings, the trade-off plots for four candidate motors, chosen from Allied Motion Megaflux series, are depicted in Fig. 6. As it is seen, the range of feasible ratios for MF0150010 is empty, i.e. this motor with any gear ratio cannot handle the requested task. Among the other three motors, MF0210025 with a comparable energy consumption is superior in terms of bandwidth. As discussed before, it is reasonable to choose the gear ratio between β_b and β_e, for example (as shown in the figure) $\beta = 12.54$.

Next, assuming that rewinding is available, we apply optimization problem (27) using NSGA-II [24] to the two best motors of the above analysis, i.e. MF0210025 and MF0310010. The obtained Pareto optimal surfaces with the population size of 120 and generation number of 4000 have

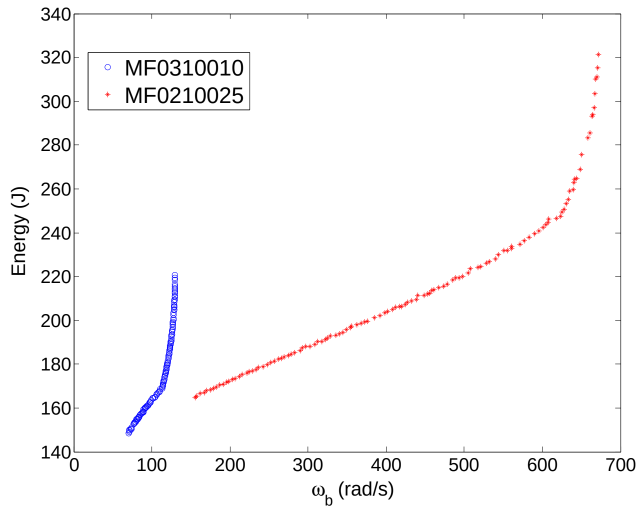

Fig. 7. Pareto fronts for two motors considered for ATRIAS

been shown in Fig. 7.

From the figure, although in comparison still MF0210025 is superior in terms of bandwidth, the interesting fact is the obvious improvement, both in bandwidth and energy. This is expected, and demonstrates the power of adding another design variable (torque constant) to the optimization, which essentially makes the actuator more suited to the task. Similar to the previous case, one can select a point from these curves as the desired design; however, in this case, each point corresponds to two variables: transmission ratio and torque constant.

VI. Conclusions

In this paper, we presented a method for choosing motor-transmission combination based on both energy efficiency and control performance. The work takes advantage of a more detailed model as well as of considering a general load case and analytical analysis of the feasible transmission ratio range, and as a result, the formulation is more comprehensive compared to the ones already in literature. Moreover, introducing bandwidth as the second objective (in addition to energy) and using the notion of Pareto dominance for choosing motors is a new concept that is a very helpful way in comparison of the motors regardless of transmission ratios. Furthermore, in the case that rewinding is available a method based on multi-objective optimization is provided for obtaining the best combinations of transmission ratio and torque constant.

Although this work attempts to consider the most general case, there are still aspects for future investigations. Further constraints (such as RMS motor torque for thermal limitations) can be included in the formulation in the normal way of optimization methods. Also, extension to cases such as variable-ratio transmissions for further optimization of the performance of the system is an attractive potential that will be studied in our future projects.

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