Toward Step-by-Step Synthesis of Stable Gaits for Underactuated Compliant Legged Robots

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Abstract— **Many control policies developed for legged robots are based on control of an underlying, simplified version of the dynamics of the robot. A good example is the Linear Inverted Pendulum Model (LIPM) which has become the standard control template for ZMP-based rigid robots. For compliant robots, this reduced order model is naturally the Spring-Loaded Inverted Pendulum (SLIP), which has proven to have many interesting traits that are potentially useful for control of full order robots. The methods proposed so far for this purpose are mainly focused on either matching the dynamics of the robot to those of SLIP, or following a SLIP-produced trajectory. These methods can be problematic, especially for underactuated systems. In the present work, we explore an opposite approach, by starting from SLIP and step-by-step constructing toward the full order robot. The goal is to detect and capture the essential stabilizing variables in the reduced order model that can potentially maintain their stabilizing effect in the full order robot, as well. Our initial investigations show that the proposed method provides excellent potentials for synthesizing stable gaits for underactuated compliant robots by use of a much simpler and more robust approach compared to the ones previously presented in the literature.**

I. Introduction

The interesting characteristics of Spring Loaded Inverted Pendulum (SLIP) model, including matching to biological observations [1], justifying different modes of locomotion [1], [2], and interesting control policies [3]–[9], has attracted the attention of the roboticists for exploiting these traits for robotic applications. Apart from fully actuated robots that tried to take advantage of SLIP policies by mimicking its dynamics [10], [11], several endeavors has been made to match the dynamics of underactuated robots to those of SLIP. Based on Singular perturbation theory, Poulakakis and Grizzle tried to embed SLIP as Hybrid Zero Dynamics (HZD) of a more complex system [12]. Hereid used an SLIP walking gait as an initial seed for HZD optimization [13]. However, the final gait obtained for the robot was completely different from that of the original SLIP. Later, he tried to match the behavior of a robot with ideal actuators (i.e. with bandwidth of infinity) to SLIP [14], but the method is not easy to be extended to realistic actuators. Wu [15] and Dadashzadeh [16] used ground reaction force control for following SLIP running policies and showed interesting simulations. However, practical application of these methods, due to relying on high-bandwidth force control, is not straightforward, and has not been checked in experiments yet.

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All of these approaches, although substantially different in methods, have a common basis: matching the dynamics of the robot to SLIP to be able to take advantage of its useful characteristics. Although, *ideally*, a fully actuated robot can be matched to a given set of dynamics (SLIP or other dynamics), the possibility of matching the dynamics of an underactuated robot to an arbitrary dynamics is not clear. Control methods for such problems usually are based on separation of time-scale and singular perturbation theory [17], which for applications such as legged robots, where the actuators are normally pushed to the limits, are not valid choices.

The subject of this paper is to investigate the above problem from a completely different angle; i.e. from SLIP to the full order robot. We aim at finding the stabilizing effects in the simplest cases and step-by-step extension of them to the more complex ones. In this way and as the first step, we will start by passive SLIP and will find a stabilizing foot placement strategy (as the sole control input for this model) based on angular momentum. Next, the system is stabilized in energy-variant space using a generalization of Schmitt and Clark's clock-driven control [8], and at the end, a torso is added to the system and the reduced order model is stabilized. After designing the controllers for the reduced order model, they are applied to a simulation of full-order dynamics of ATRIAS, a bipedal underactuated robot, and the results are assessed and discussed.

II. Step-by-Step Control Design

A. Stabilization of SLIP Using Angular Momentum-Based Foot Placement

The most basic model that a compliant legged robot can be reduced to is SLIP model (Fig. 1). Essentially, SLIP consists of a point mass representing the body, and one (for hopping or running) or two (for walking) massless springy legs. There is no actuator in SLIP and thus it is energetically conservative. Therefore, the only control input is foot placement. Inspired by this fact, several stabilizing controllers have been studied in the literature, mainly for SLIP running. Saranli [3] proposed a deadbeat controller which by use of inverse dynamics, transfers the states of the system to the desired ones in one step. Seyfarth et al. [4] showed that a leg retraction policy in the second half of the flight phase can help stabilization. As well, Altendorfer et al [6] mathematically proved that symmetry of the leg angles in take-off and touchdown results in stabilization. Based on this, Schmitt [7] suggested a feedback for the leg angle of attack to further stabilize the gait. More recently, Ernst et al. [5] proposed a swing leg trajectory for running on rough train which leads to maintaining an invariant apex speed.

Whereas SLIP hopping and running controllers are relatively well-studied, SLIP walking has only attracted attention in recent years [1]. Due to intrinsic difference between hopping/running and walking, most of the SLIP running controllers are not applicable to walking. As a result, Martinez [2] in his study of SLIP walking used the constant leg angle of attack as his foot placement policy. As he showed, most of the gaits with this policy are unstable, and hence he used the number of steps to falling as a measure of stability. More recently, Vejdani [18] discovered that extracting foot placement conditions from the stance leg parameters can lead to stabilization of gaits. This is especially interesting, in that, in a sense, it can be related to HZD, wherein the actuated states, including swing leg DoFs, are written as functions of stance leg angle [19]. Although this dependence in HZD has been used as a way for coming up with time-invariant holonomic "virtual constraints", the stability of SLIP walking with this policy can be regarded as a determining factor for highly stable and robust walking gaits exhibited by compliant robots controlled by HZD [20],[21].

Inspired by this, we turn our attention to the physical mechanisms through which foot placement stabilizes passive walking. It is well-known that for a system with point feet and no ankle-actuation, the rate of the angular momentum around the stance foot cannot be regulated. In other words, after Vertical Leg Orientation (VLO), even in the presence of actuators, the absolute value of angular momentum always increases, until the other foot touches the ground. In this regard, foot placement can be viewed as a discrete regulation tool for the parameter that otherwise cannot be changed in the continuous dynamics, and preventing it from increasing more than a specific threshold (which indeed depends on the gait). In particular, the "controlled" switching manifold (from single-support to double-support) can be defined as:

$$\mathcal{S} := \left\{ z \in \mathcal{Z}_{\bar{E}}^{+} \mid \; H(z) = H_0 \right\}, \tag{1}$$

where z and $\mathcal{Z}_{\bar{E}}^{+}$ are state vector and state manifold (with a constant energy of $\bar{E}$) of the second half of the stance, respectively; H is the angular momentum around the stance foot, and H_0 is the predefined angular momentum at which the phase switch occurs. Note that since the legs are massless, the switching manifold does not have dynamics.

Unfortunately, since the dynamics of SLIP are not integrable [22], analytical investigations for stability (except for special cases such as [6]) are not possible. As a result, almost all of the studies related to SLIP model rely on numerical analyses. Therefore, in order to assess the stability of the proposed control, we compute the maximum absolute values of the eigenvalues of the Poincaré section (defined at VLO) of symmetric gaits[1] for a specific energy level with three different switching control laws; namely: angular momentum, stance leg angle, and swing leg's angle of attack (Fig. 2).

[1] In the present work, for the study of the reduced order models, we limit ourselves to symmetric gaits; i.e. the gaits with symmetry around VLO. In such gaits the vertical velocity at VLO and at the middle of the double-support phase, due to symmetry, becomes zero. However, the proposed methods are not limited to this type of gaits.

As expected, and was previously shown in [2], constant angle of attack is an unstable policy and the associated eigenvalues are more than 1. The comparison between angular momentum and stance leg angle policies is more interesting. The two policies show similar trends, but the eigenvalues of angular momentum policy are almost consistently smaller. Furthermore, if we define the preferred gait in a specific energy level as the gait with the smallest eigenvalue, as far as our investigations show, the preferred gait of SLIP with the angular momentum policy is always more stable than the stance leg angle policy's preferred gait.

With the above argument, we choose the angular momentum as the determining parameter for foot placement to stabilize passive (energy-conservative) SLIP walking. In the next section, we extend the stabilization to non-conservative SLIP.

B. Stabilization in the Presence of Energy Variations

Although passive SLIP has proven to capture the essential characteristics of animal locomotion, its basic assumption for conservation of energy is a shortcoming for obtaining deeper insights both in understanding animal locomotion using this reduced order model and its application to robot control. Indeed, in any real-world locomotion system (biological or robotic) energy varies due to various factors such as uneven ground, impacts, internal losses, etc. Passive SLIP limits the study only to $\mathcal{Z}_E$, the energy-conservative submanifold of the manifold of all possible states, $\mathcal{Z}$. Although it is highly useful to simplify the analysis for obtaining insights (as we did in the previous section), when the states are considered in $\mathcal{Z}$, they can be at best neutrally stable: if the system is kicked to a new energy level, there will be no means to return it to the original set of states.

To resolve this problem, in general, two different approaches can be taken. In the first approach, a feedback low is designed to always maintain the system in a constant energy level. Passivity-based controllers which are well-known in various fields of robotics [23] are examples of such controllers. Poulakakis [12] used this method to design a controller for SLIP with leg length actuation ("Energy-stabilized SLIP, or E-SLIP), and based on that, Dadashzadeh [16] designed a controller to match the force profile of a robot to that of the passive SLIP.

Although the stability of this method is certainly guaranteed through an associated Lyapunov function, the need for extensive and high-speed sensory information from one side, and our objective for finding self-stable periodic orbits from the other side, motivate us to seek another approach.

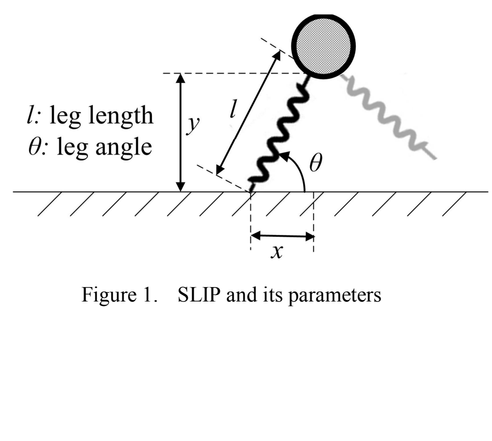

Figure 1. SLIP and its parameters

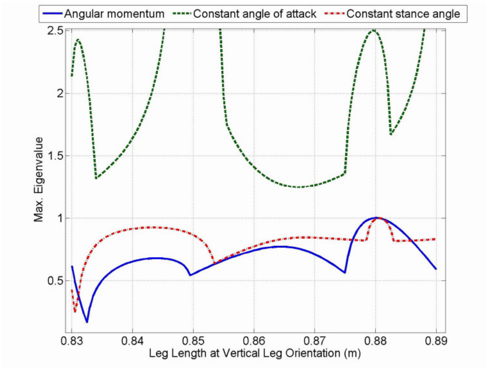

Figure 2. Eigenvalues of the Poincaré section for SLIP walking with three different foot placement policies

In the second approach, the energy is also changed along the periodic orbit. In other words, the periodic orbit is not traverse to any energy-conservative submanifold. However, since the energy (as any other variable in the system) must be periodic, it means that the actuators add and remove a fixed amount of energy in one cycle of the nominal gait. Specifically, in order to regulate this energy, the displacement of the leg length actuator during stance can be written as a function of q, a function of state variables and/or time. We have:

$$\dot{E} = F\dot{l}_m = k(l_m - l)\dot{l}_m \tag{2}$$

where F is the force, k is the spring constant, and l_m is the neutral length of the spring which is set by the actuator.

From (2), in order to have $\Delta E = \int_0^{t_{st}} \dot{E}\, \mathrm{d}t = 0$ (with t_{st} as the duration of stance phase), and noting that for symmetric gaits F is an even function around the midstance, $\dot{l}_m$ must be an odd function, and consequently, the condition of l_m being an even function will be sufficient for adding and removing an identical amount of energy. To find another condition on l_m, we have:

$$\mathrm{sgn}\left[\frac{\mathrm{d}t_{st}}{\mathrm{d}\Delta E}\right] = \mathrm{sgn}\left[\frac{\mathrm{d}\Delta E}{\mathrm{d}t_{st}}\right] = \mathrm{sgn}\left[\dot{E}(t_{st})\right] = \mathrm{sgn}\left[\dot{l}_m|_{t=t_{st}}\right] \tag{3}$$

Now, since typically with the increase (decrease) of the energy from the nominal value stance time becomes shorter (longer), the amount of energy required to regulate the system back to the nominal value, ΔE, is negative (positive) and thus in any case we need to have: $\dot{l}_m|_{t=t_{st}} \geq 0$, and consequently, due to symmetry: $\dot{l}_m|_{t=0} \leq 0$ (where $t = 0$ indicates the start of stance phase). Note that the above analysis is not limited to stance time, and thus any other variable, q, which is sufficiently monotonic with change of energy, can be chosen as the independent variable defining l_m. In such cases, $\mathrm{sgn}\left[\frac{\mathrm{d}q_{st}}{\mathrm{d}\Delta E}\right] = \pm\mathrm{sgn}\left[\frac{\mathrm{d}l_m}{dq}|_{q=q_{st}}\right]$, depending on whether q_{st} is increasing or decreasing with energy.

From a biological perspective, Schmitt and Clark [8] suggested a "clock-driven" formulation as a special case of what mathematically derived in the above. In their formulation:

$$l_m = l_0 - l_{dev} \sin\frac{\pi t}{t_{st}} \tag{4}$$

and through numerical analyses for all values of l_{dev}, they confirmed what we obtained in the above, i.e. for stabilization we need to have: $l_{dev} > 0$.

The above argument enables us to choose a more suitable function for l_m, particularly with an eye on the practical application on the robot. Note that the signs of $\dot{l}_m$ at the beginning and the end of stance, as obtained in the above, is exactly opposite of what is required for smooth transition between stance and swing/flight phases. For instance, at touchdown, if $\dot{l}_m < 0$, the leg length will be decreasing, which in a real-world application, may cause the leg miss the ground. Therefore, we generalize the Schmitt-Clark formulation as:

$$l_m = l_0 - l_{dev}\left[\cos\left(c\pi\left(\frac{t}{t_{st}} - \frac{1}{2}\right)\right) - \cos\left(\frac{c\pi}{2}\right)\right] \tag{5}$$

where $1 \leq c \leq 2$ is a constant parameter to be selected. Note that Schmitt-Clark's formula is a special case of (5) with $c = 1$. With this new formulation, c can be chosen such that after a prespecified time in the second half of stance, $\dot{l}_m$ becomes negative and the leg starts to retract, resulting in smooth transition to the swing phase. However, as adding and removing energy (as discussed before) is performed in a less aggressive way, the energy stabilization effect is somewhat weakened, which makes a trade-off between stability and smoothness.

C. *Torso Stabilization*

In the next step toward synthesis of a stable gait for a full-order robot, we add a torso and a leg angle actuator to the energy-stabilized system of the previous section (Fig. 3).

In the previous works related to this subject, torso is usually stabilized at a fixed angle using the leg angle actuator [12], [16]. However, a closer look at the problem reveals that applying leg angle torque to a torso with the center of mass above the hip takes positive work in the first half of stance and negative work in the second half from the leg angle actuator. This is exactly opposite to the stabilization of energy designed in the previous section, and as a result, will be at odds with the stability. Indeed, using this policy, all gaits we found were at best marginally stable (i.e. maximum eigenvalues were around 1). Therefore, since there is no requirement for imposing such a strict condition, we relax the condition and allow the torso to move in a predefined range. Thereby, the inertia of the torso can be utilized appropriately for helping stabilization.

As mentioned before, we are interested in symmetric gaits and thus we limit the trajectories to these cases. For symmetric gaits the torso angle (from the vertical direction), q_T must change in a symmetric interval $[-q_{dev}, q_{dev}]$ and as well, we must have: $q_T = 0$ at midstance. This motivates the use of trajectories in the form of:

$$q_T = q_{dev} \sin \Omega t, \tag{6}$$

wherein time is measured from the midstance. Note that $\Omega \neq \frac{2\pi}{t_{st}}$, because in general, in the double-support phase there is an overlap between the timing of the two legs. $\Omega = \frac{2\pi}{t_{st}}$, if and only if the double-support phase is instantaneous. The above trajectory defines a holonomic virtual constraint on the dynamics of the system. However, as it has been chosen as a time-dependent function, the constraint is not scleronomic as it is the case of HZD constraints. As it was mentioned for the energy stabilizing function in the previous section, a suitable state variable instead of time may accommodate the same result, as well.

Using the above trajectory, we found that relatively small positive deviation amplitudes (q_{dev}) are sufficient to stabilize the system. Thus the trade-off will be between the range of oscillations of the torso, and the stability.

To design a controller for determining the leg angle torque to track the given trajectory we use input-output feedback linearization [24]. Take:

$$q_{rom} = \left[x, y, q_T\right]^T, \tag{7}$$

as the generalized coordinates for the reduced order model. The Lagrange equations for the reduced order model can be written as:

$$D_{rom}\ddot{q}_{rom} + P_{rom}(q_{rom}, \dot{q}_{rom}) = F_l + Bu_\theta \tag{8}$$

where u_θ is the leg angle torque input. Also,

$$F_l = k(l_m - l)\begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} \\ \frac{y}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix} \tag{9}$$

is the spring force, and

$$B = \begin{bmatrix} \frac{-y}{x^2+y^2} \\ \frac{x}{x^2+y^2} \\ -1 \end{bmatrix} \tag{11}$$

Taking $z = [q_{rom}, \dot{q}_{rom}]$ the dynamic equations can be written in the affine form:

$$\dot{z} = f(z) + g(z)u_\theta \tag{12}$$

where

$$f(z) = \begin{bmatrix} \dot{q}_{rom} \\ D_{rom}^{-1}(F_l - P) \end{bmatrix} \tag{13}$$

and

$$g(z) = \begin{bmatrix} \mathbf{0} \\ D_{rom}^{-1}B \end{bmatrix} \tag{14}$$

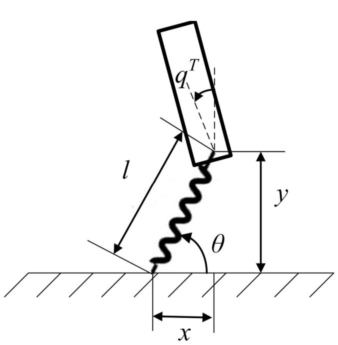

Figure 3. SLIP with torso, and its parameters

In this form, the control law

$$u_\theta = \frac{1}{\mathscr{L}_g\mathscr{L}_f q_T}\left(-k_1\left(q_T - q_T^d\right) - k_2\left(\dot{q}_T - \dot{q}_T^d\right) + \ddot{q}_T^d - \mathscr{L}_f^2 q_T\right) \tag{15}$$

with k_1 and k_2 positive constants, renders q_T stably to q_T^d, the desired trajectory for the torso pitch.

For future reference, note that the inertia matrix of the reduced order model has the following form:

$$D_{rom} = \begin{bmatrix} m & 0 & -ml_T\cos q_T \\ 0 & m & -ml_T\sin q_T \\ -ml_T\cos q_T & -ml_T\sin q_T & I_T + ml_T^2 \end{bmatrix} \tag{16}$$

with m being the mass, and I_T and l_T the inertia of the torso and the distance between its center of mass to the hip, respectively. As it is evident, the inertia matrix has a very simple form and its inversion does not impose much computational cost.

The above controller has been formulated for single-support, but extension to double-support is straightforward.

III. Application of the Proposed Method to a Full-Order Robot

A. ATRIAS

ATRIAS (Fig. 4) is a human-scale bipedal robot with the ability of 3-D locomotion, designed and built in Dynamic Robotics Laboratory of Oregon State University [25]. ATRIAS' leg mechanism has been specifically designed to capture the essential characteristics of the spring-mass model (Fig. 5), including light legs, springs that can be effectively act in leg length direction and store a significant amount of energy, and actuators separated from impact through these springs. These traits make ATRIAS a good choice for implementation of the control ideas presented in the previous section.

B. Dynamic Modelling

ATRIAS in 2-D and with point feet, as it is considered in this study, has 11 DoFs and 4 actuators. The generalized coordinates are taken in the form of:

$$q = [x, y, q_T, q_l, q_m]^T, \tag{17}$$

Figure 4. ATRIAS, a bipedal compliant robot

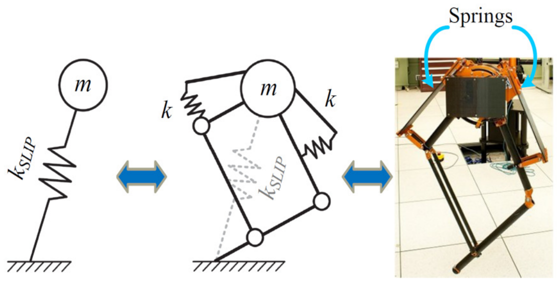

Figure 5. ATRIAS has been designed to match SLIP as closely as possible.

where x and y are the horizontal and vertical coordinates of the hip with respect to the foot, $(q_l)_{4\times1}$ contains the leg coordinates, $(q_m)_{4\times1}$ the corresponding motor coordinates, and q_T the torso angle, respectively.

Defining coordinates capturing only the rigid body structure of the robot (i.e. removing the series elastic actuators' dynamics) as:

$$q_{rb} = [x, y, q_T, q_l]^T, \tag{18}$$

Now, the Lagrange equations take the form of:

$$\begin{cases} D(q_{rb})\ddot{q}_{rb} + C(q_{rb}, \dot{q}_{rb})\dot{q}_{rb} + G(q_{rb}) = \tau_s + J^T F \\ I_m\ddot{q}_m + b_m\dot{q}_m + k(q_m - q_l) = u \end{cases} \tag{19}$$

where I_m and b_m are actuator inertia and damping, respectively, k is the spring constant, and

$$\tau_s = k\begin{bmatrix} \mathbf{0} \\ q_m - q_l \end{bmatrix} + b\begin{bmatrix} \mathbf{0} \\ \dot{q}_m - \dot{q}_l \end{bmatrix} \tag{20}$$

is a vector containing the spring torques (which are the interface between actuators and the rigid body mechanism), and J^T is the Jacobian transferring the ground reaction forces, F, to the rigid body coordinates. u contains the motor torques.

C. Phases of Walking as a Hybrid System and Impact

Walking is a hybrid process that can be expressed by the two distinct phases, namely single-support ("*ss*") ,and double-support ("*ds*"). The domains and guards for the two phases of this hybrid process are given by:

$$\begin{cases} \mathscr{D}_{ss} = \{(q, \dot{q}, u) \mid y_f \geq 0 \ \wedge F_y = 0\} \\ \mathscr{G}_{ss} = \{(q, \dot{q}) \mid y_f = 0 \ \wedge \dot{y}_f < 0\} \end{cases} \tag{21}$$

and

$$\begin{cases} \mathscr{D}_{ds} = \{(q, \dot{q}, u) \mid y_f = 0 \ \wedge F_y \geq 0\} \\ \mathscr{G}_{ds} = \{(q, \dot{q}, u) \mid F_y = 0 \ \wedge \dot{y}_f > 0\} \end{cases} \tag{22}$$

where y_f is the corresponding foot's height from the ground and F_y is its vertical force. $\mathscr{G}_{ds}$ does not have an impulse effect. i.e. the states of the system do not jump. But $\mathscr{G}_{ss}$ is associated with a mechanical impact that induces a jump in the velocities:

$$\dot{q}^+ = \Delta(q, \dot{q}^-) \tag{23}$$

where $\dot{q}^-$ and $\dot{q}^+$ are velocities immediately before and after impact, respectively. Touchdown is modeled as an inelastic impact wherein the absolute velocity of the foot immediately comes to zero. For details of the model, see [26], [27].

D. Control

The controllers proposed in the previous sections (Eqs. (5) and (15)) set leg length forces and leg angle torques. A simple Jacobian mapping can transfer these values to joint torques:

$$\tau_a = J_l^T \begin{bmatrix} F_l \\ u_\theta \end{bmatrix} \tag{24}$$

By exploiting the fact that the damping of the springs is small, take:

$$q_m^d = {}^{\tau_a}\!/_k + q_l \tag{25}$$

Thereby, the control implementation is converted to motor position control, which can be performed by Proportional-Derivative (PD) controllers with feedforward terms:

$$u = K_p(q_m^d - q_m) + K_d(\dot{q}_m^d - \dot{q}_m) + k(q_m - q_l) + b(\dot{q}_m - \dot{q}_l) \tag{26}$$

The PD gains are the values that were verified in our experiments on ATRIAS (cf. for example [13]).

Note that, as briefly mentioned before, due to requirement for inversion of the inertia matrix, the feedback linearization controller terms are typically so large that make the online usage impractical. As such, in HZD, for example, feedback linearization is used only for offline optimizations [27]. However, since we apply feedback linearization only in the reduced order model level, the terms are simple (see (16)) and the controller is expected to be readily implemented to the real robot without difficulty.

The two controllers mentioned in the above fully define the actuator inputs of the stance leg. For the swing leg, the two actuators, by use of the same PD controllers as before, track a predefined 2-D foot trajectory which sets the end of swing (i.e. foot placement) according to the proposed angular momentum policy. The touchdown angular momentum, H_0, is obtained from the periodic orbit for the (symmetric) gait of the reduced order model.

E. Results and Discussion

Based on the method proposed thus far, a stable walking gait with the forward speed of 0.75 m/s was obtained for the reduced order model with $l_0 = 0.9$ m, $l_{dev} = 1.5$ cm, and $q_{dev} = 2.5$ deg. These parameters, together with the touchdown angular momentum, H_0, fully define the walking of the robot. Note that, as discussed before, the goal is not to match the dynamics of the robot to those of the reduced order model. Rather, the purpose is to investigate the stabilization effect of the controllers proposed for a reduced order model on a system with substantially more complex dynamics. Figs. 6 to 9 depict the simulation results.

Fig. 6 shows the phase portrait for forward motion of ATRIAS vs. that of the reduced order model. The two systems start from the same initial condition, but as it can be observed from the figure, ATRIAS deviates from the reduced order model's periodic orbit and after a few steps converges to a new, lower-speed periodic orbit. It is expected, as many sources of energy loss, including touchdown impact, frictions, dampings, etc., which have been considered in the ATRIAS simulation are absent in the reduced order model. As a result, in order to balance between the losses and the energy given by the controllers, the system settles on a lower-energy gait. The gait is a period-2 orbit, i.e. the states of the left and the right leg evolve differently, which is due to the fact that the frequencies taken from the reduced order model (for torso pitch and leg length actuation trajectories, cf. (5) and (6)) do not necessarily match in the new periodic orbit to result in a period-1 limit cycle.

Fig. 7, the phase portrait of the torso pitch (initial steps have been removed for the sake of clarity), is also very interesting, in that it shows an asymmetric deviation from the reduced order model. Specifically, the positive torso pitch angles (backward pitch) are close to the ones from the reduced order model, whereas the negative ones show larger deviations. This can also be explained as a symptom of the lower energy gait discussed in the above. As the touchdown is set by a predefined value of angular momentum, and since the speeds are lower in the ATRIAS gait, the torso pitches forward until the touchdown condition is satisfied and the ground reaction force of the new stance leg returns the torso to the straight upward angle. Note that still the absolute value of the angle is less than 7° (see Fig. 9, too). Also, note that the fact that the limit cycle is period-2 is more conspicuous in this figure.

The force profiles, as depicted in Fig. 8, are other interesting characteristics. Note that the force profile of the reduced order model, due to deviations from passive SLIP is not the standard “double-peak” SLIP walking profile (cf. [1]) anymore. Nonetheless, the robot has converged to a double-peak profile, presumably due to energy losses. Also, note that the robot’s peak force, due to moving in a lower speed, is smaller.

As can be inferred from these plots, despite all the discrepancies between the reduced order model and the full order robot model, and with control parameters taken exclusively from the reduced order model, the robot converges to a stable periodic orbit. We believe this is due to the intrinsic stabilization effect of the three-step controller (angular momentum foot placement, energy stabilization, and torso trajectory). Each of these controllers was designed based on an essential stabilization trait that was expected to remain effective for the full order robot. The interesting characteristic is that changing the associated parameters (for example the touchdown angular momentum, as shown in Fig. 2) to a large extent does not affect the stability; the system simply converges to another periodic orbit (possibly asymmetric or period-2) and is stabilized around that one, and this very phenomena has worked here to stabilize ATRIAS. From this perspective, the proposed controller is similar to the symmetry-based controller of Hyon and Emura [28], where they found a globally invariant control logic stabilizing in a region of stable orbits.

IV. Conclusion

A new control method for obtaining stable gaits for compliant underactuated robots was proposed. The approach was based on step-by-step stabilization, starting from the most basic reduced order model (i.e. passive SLIP) and then adding energy and torso stabilization to it. The method was applied to the full dynamic model of ATRIAS and the robot successfully converged to a stable walking gait.

In the design of the method, we intended to avoid case-specific approaches (such as trajectory optimization), and tried to capture the essential dynamic parameters that can affect the stability of walking of compliant/spring-mass robots. The first results, as presented in this paper, show great promise for extension to more complex cases, such as walking on uneven and variable-impedance grounds and especially 3-D walking, where other approaches for underactuated dynamic walking such as HZD and transverse linearization [29] may suffer from instability [30], or huge computation costs, respectively. Although optimization algorithms have not been used for this research, they can definitely be employed for better performance of the method, especially, since the number of parameters are very low (compare, for example, with the number of Bézier curve parameters, optimized in HZD [20]). As well, aspects such as investigation of state-based trajectories instead of time-based ones (as, for instance, observed and discussed in [31], and briefly formulated in section II.B of the present work), and also studying and exploiting asymmetric gaits for possible advantages in efficiency and stability (especially in the cases with larger impacts) are potential questions to be answered in simulations and experiments in the next steps.

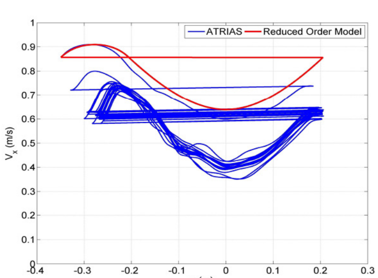

Figure 6. Phase portrait of forward motion, ATRIAS vs. the reduced order model. Note that the robot converges to a stable gait, regardless of tracking.

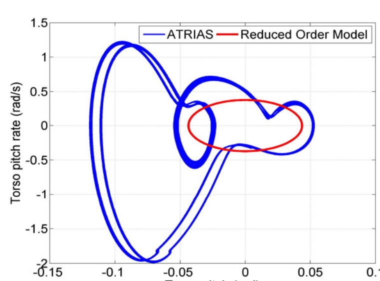

Figure 7. Phase portrait of torso, ATRIAS vs. the reduced order model

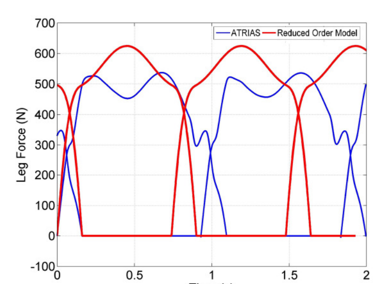

Figure 8. Leg forces, ATRIAS vs. the reduced order model

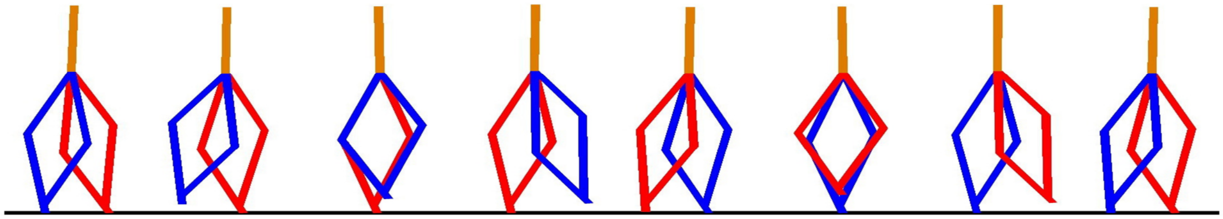

Figure 9. Snapshot of ATRIAS walking in a full stride (two steps)

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