Effect of convective cooling on frictionally excited thermoelastic instability

Y.B. Yi*, A. Bendawi

Department of Mechanical and Materials Engineering, University of Denver, Denver, CO 80208, USA

ARTICLE INFO

ABSTRACT

The effect of convective cooling on thermoelastic instability is evaluated using a finite element analysis. This is achieved by inserting a thermal convection term to the frictional heat generation in the formulation. It has been found that the convection or radiation heat dissipation can stabilize the thermal–mechanical feedback process, leading to a raised critical sliding velocity. Two representative models for brake and clutch systems are studied. The computational results reveal that the effect of thermal convection on the critical sliding speed is significant for liquid cooling (e.g. water, lubricants), but negligible for air convection. With the practical range of the convection coefficient estimated from the fundamental heat transfer theories, the critical speed in the presence of convection can be raised by two to three times as much as the original value. However, the wave number for the lowest critical speed remains almost unchanged regardless of the convective dissipation. The comparisons between linear and quadratic finite element interpolations are also made via a set of convergence studies. The results show that implementing quadratic elements in the friction layer has an obvious advantage over linear elements due to the rapid oscillations of the temperature across the thermal skin layer. This is particularly important in future studies when the problems in higher dimensions are of interest.

1. Introduction

It is well known that *thermoelastic instability* (or TEI) occurs in high speed frictional sliding systems such as disk brakes or clutches [1]. If the sliding speed exceeds a critical value, a small perturbation in the system can lead to an exponentially growing temperature or contact pressure due to the thermal–mechanical feedback. To predict the growth rate of temperature/pressure at a given sliding speed or the critical value of the speed above which the system becomes unstable, the perturbation method, sometimes in conjunction with the finite element formulation, is applied in a variety of applications. This typically leads to an eigenvalue equation in the matrix form, from which the critical sliding speed of the system can be found by searching for the lowest speed at which the leading mode has a positive growth rate.

Since the mechanism of TEI was first discovered by Barber in 1969 [2], both the analytical [3,4] and numerical [5,6] approaches have been widely used in the stability analyses. For the latter, the demand for extensive numerical iterations may impose a major hurdle especially on transient analyses. In automotive applications involving axisymmetric disk-like geometries, the Fourier reduction method has proven an efficient way to overcome this difficulty [7]. Linear perturbation solutions [8] are sought that vary sinusoidally in the circumferential direction, resulting in an eigenvalue problem defined on the cross-sectional thickness domain.

The effects of various geometric configurations [9,10] as well as material properties [11,12] on the stability boundaries of the TEI phenomenon were investigated. However, traditionally the conductive heat transfer inside the moving solids was the only concern in most of the formulations. Other sources of heat dissipation including convective cooling and radiation, were nearly always neglected. Convective cooling, in fact, is a major heat dissipation mechanism and plays an important role in brake and clutch systems. Without conductive cooling, temperatures may exceed 1000 °C on some brake disk surfaces in the friction contact region [13], and in some extreme situations can reach as high as 2000 °C for carbon–carbon composite multidisk brakes in aircraft applications [14].

It is well known that convective cooling is important in lowering the maximum bulk temperatures of brake or clutch systems, the effect of thermal convection on the TEI phenomenon, however, is much less studied, mainly because of the fact that the theory of TEI is concerned about the temperature variation rather than the steady state temperature level. It was believed that the majority part of heat exchange in the system occurs between the alternating hot and cold regions of the solids, as opposed to the heat transfer taking place on the fluid–solid interface in convective cooling. Due to the high sliding speed in the automotive applications, the rate of heat exchange inside the solids is

* Corresponding author. Fax: +1 3038714450.
E-mail address: yyi2@du.edu (Y.B. Yi).

typically much faster than the convective cooling, especially when the system is cooled by air. This is because air has a relatively low convection coefficient. Some heavy duty clutches and brakes used in naval and aerospace applications, however, are liquid cooled with the cooling water contained in the water jackets [15]. Transmission fluids used in wet clutch systems may also play an important role in convective cooling. While the coefficient of air convection rarely exceeds 200 W/m² K, it is not unusual to have a convection coefficient for liquid cooling greater than 10,000 W/m² K [16]. It is noticed that some pioneering work was already made by Zagrodzki regarding the effect of Newtonian cooling on TEI [17], however a systematic investigation on the phenomenon, e.g. how and to what extent the convective cooling rate can affect the stability boundaries of TEI, is still unavailable in the literature. Additionally thermal radiation may contribute to the overall heat dissipation. It will be shown later that the effect of radiation is analogous to that of convection by introducing an equivalent convection coefficient, which can be readily obtained from the Stefan–Boltzmann Law.

2. Basic methodology

2.1. Heat transfer equation

The two-dimensional heat conduction equation for the sliding bodies in the fixed frame of reference x and y shown in Fig. 1 can be written as

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \left(\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial y}\right) = 0 \tag{1}$$

where k is the thermal diffusivity. If the sliding takes place in the y-direction, then one can assume an exponentially growing perturbation solution of the following form

$$T(x,y,t) = T_0(x,y) + \Re\{e^{bt+jmy}\Theta(x)\} \tag{2}$$

where T_0 is the steady state solution, b is a complex exponential growth rate and m is the wave number defined as the number of oscillations for perturbation within a length of 2π. Note that m can take any positive real value and it has a unit m^{-1}. $\Re$ represents the real part of a complex number. Substitution of Eq. (2) into Eq. (1) yields

$$k\frac{\partial^2 \Theta}{\partial x^2} - km^2\Theta - (jmV + b)\Theta = 0 \tag{3}$$
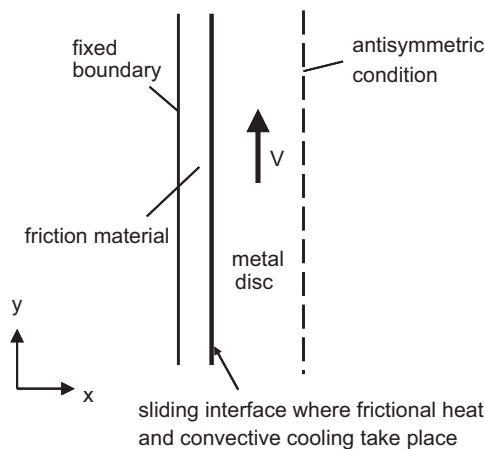

Fig. 1. Schematic of the computational model.

Applying the Galerkin finite element formulation leads to a matrix equation in the form

$$(\mathbf{K} + \mathbf{R} + b\mathbf{H})\Theta + \mathbf{Q} = 0 \tag{4}$$

where $\mathbf{Q}$ is the nodal heat flux and the rest are the matrices evaluated from the aggregation of the following elemental matrices

$$\mathbf{K_e} = \int k\left(\frac{\partial \mathbf{N}}{\partial x}\frac{\partial \mathbf{N}^T}{\partial x}\right)dx \tag{5}$$

$$\mathbf{R_e} = \int (km^2 + jmV)\mathbf{N}\mathbf{N}^T dx \tag{6}$$

$$\mathbf{H_e} = \int \mathbf{N}\mathbf{N}^T dx \tag{7}$$

where $\mathbf{N}(x)$ is the shape function.

2.2. Frictional heat generation

The rate of frictional heat generation on the contact interface of the sliding bodies is given by

$$\mathbf{Q}_{fric} = fV\mathbf{\Phi}\mathbf{P} \tag{8}$$

where $\mathbf{Q}$ is the nodal heat generation (heat flux), $\mathbf{P}$ is the nodal contact pressure defined at the contact nodes only, and f is the coefficient of Coulomb friction. $\mathbf{\Phi}$ is a coefficient matrix constructed in the following way

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}^T \tag{9}$$

2.3. Convective heat dissipation

Assuming a constant heat transfer coefficient h over the entire contact area, the convection heat loss expressed in the matrix form is then given by

$$\mathbf{Q}_{conv} = h\Theta \tag{10}$$

where $\mathbf{Q}_{conv}$ is defined as the nodal heat loss due to convection.

Considering the net heat generation on the contact interface, Q, we have

$$\mathbf{Q} = \mathbf{Q}_{fric} - \mathbf{Q}_{conv} = fV\mathbf{\Phi}\mathbf{P} - h\Theta \tag{11}$$

Please note that the terms in the above equation contain the variable components only. The constant quantities such as the ambient temperature in the convective heat dissipation satisfy the steady state heat transfer equations and therefore do not appear here. It should also be pointed out that this formulation does not apply to the clutch system described in reference [15] where water jackets cool the back side of the metal plate rather than the sliding interface. However, the temperature variation on the back side is generally much lower than that on the sliding interface, and therefore it is believed that the effect of convective cooling on TEI is less important in that situation.

2.4. Thermoelasticity

For plane strain problems, the constitutive law for thermoelasticity is

$$\sigma = \mathbf{C}e - \mathbf{D}T \tag{12}$$

where

$$\mathbf{C} = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & 1-v/2 \end{bmatrix} \tag{13}$$

$$\mathbf{D} = \frac{E\alpha}{(1-2v)}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \tag{14}$$

where v is Poisson's ratio, E is the elastic modulus, α is the coefficient of thermal expansion and e is the strain, which can be expressed in the matrix form in terms of the nodal displacement vector $\mathbf{U}$. That is

$$e = \mathbf{BU} \tag{15}$$

By assuming the displacement components in the perturbation forms

$$u_x(x,y,t) = u_{x0} + \Re(e^{bt+jmy}U_x) \tag{16}$$

and

$$u_y(x,y,t) = u_{y0} + \Re(e^{bt+jmy}U_y) \tag{17}$$

One can obtain an additional matrix equation

$$\mathbf{L\,U} - \mathbf{G\Theta} = \mathbf{\Phi P} \tag{18}$$

where $\mathbf{G}$ is a square matrix and $\mathbf{L}$ is a rectangular matrix. The elemental matrices for $\mathbf{L}$ and $\mathbf{G}$ are defined by

$$\mathbf{L_e} = \int_\Omega \mathbf{B}^T \mathbf{CB}dx$$

$$\mathbf{G_e} = \int_\Omega \mathbf{B}^T \mathbf{DN}dx \tag{19}$$

and

$$\mathbf{B}_i(m,x) = \begin{bmatrix} \frac{d\mathbf{N}_i}{dx} & 0 \\ 0 & m\mathbf{N}_i \\ -m\mathbf{N}_i & \frac{d\mathbf{N}_i}{dx} \end{bmatrix} \tag{20}$$

Here $\mathbf{N}_i$ represents the shape function of the ith node in a given element. Rearranging Eq. (18) in the partitioned form yields

$$\begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix}\mathbf{U} - \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix}\mathbf{\Theta} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\mathbf{P} \tag{21}$$

where $\mathbf{I}$ is the identity matrix of order n_c, the number of the contact nodes. Therefore

$$\mathbf{L}_1\mathbf{U} - \mathbf{G}_1\mathbf{\Theta} = \mathbf{P}$$
$$\mathbf{L}_2\mathbf{U} - \mathbf{G}_2\mathbf{\Theta} = 0 \tag{22}$$

In the above formulation, the nodal vectors are partitioned according to whether they belong to the contact nodes. This is because the contact node pairs share the same normal displacements, and also because the frictional heat is presumably generated at the contact nodes. Eq. (22) immediately yields

$$\mathbf{U} = \mathbf{L}_2^{-1}\mathbf{G}_2\mathbf{\Theta}$$
$$\mathbf{P} = (\mathbf{L}_1\mathbf{L}_2^{-1}\mathbf{G}_2 - \mathbf{G}_1)\mathbf{\Theta} \tag{23}$$

2.5. Eigenvalue equation

Combining Eqs. (4), (11) and (23) yields the following eigenvalue equation

$$\mathbf{M\Theta} = b\mathbf{H\Theta} \tag{24}$$

where

$$\mathbf{M} = fV(\mathbf{G}_1 - \mathbf{L}_1\mathbf{L}_2^{-1}\mathbf{G}_2) - \mathbf{K} - \mathbf{R} - h \tag{25}$$

Note that it is a standard eigenvalue equation without considering the inertial effects.

2.6. Convective heat transfer coefficient

In automotive applications it is generally believed that both laminar and turbulent flows are viable depending on design and operational parameters. For simplicity, here we neglect the laminar flow regions and assume an idealized situation in which the turbulent flow covers the entire sliding surface. For an external, turbulent flow over an isothermal flat plate, the local convection heat transfer coefficient can be approximated from the Reynolds Analogy [16] as

$$Nu_x = 0.0296\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}, \quad \text{where } 0.6 \le \mathrm{Pr} \le 60 \tag{26}$$

In the above approximation, Pr is the Prandtl number, and Nu_x is the Nusselt number defined by

$$Nu_x = \frac{h_x x}{K_f} \tag{27}$$

where h_x is the local convection coefficient and K_f is the thermal conductivity of the fluid inside the boundary layer. The Reynolds number is defined by

$$\mathrm{Re}_x = \frac{\rho U x}{\mu} \tag{28}$$

where Re_x is the Reynolds number, ρ is the fluid density, μ is the fluid viscosity, U is the linear velocity component parallel to the plate. Assuming a typical clutch or brake disk diameter as the characteristic length, and the maximum sliding speed to be 5000 rpm, it has been estimated that in the presence of water, the maximum convection coefficient can reach 20,000–30,000 W/ m^2 K, and this result is in fact quite consistent for a wide range of the characteristic length of the sliding bodies. The result is also consistent with the order of magnitude of the convection heat transfer coefficients reported in the literature regarding the wet clutch applications [18]. Note that in the above approximation, linear motion at a constant speed and parallel flow over a flat plate were assumed. In reality, however, it may vary depending on the flow conditions and geometric configurations such as grooves. Nonetheless it is expected that the actual result should have the same order of magnitude. If the transmission lubricant instead of water is used as the coolant, the high viscosity of the lubricant can bring down the Reynolds number and hence reduce the overall convection coefficient. In addition, in the above estimation the external flow condition was assumed, which is actually not the case when the lubricant is confined between the two sliding surfaces. Hence the above result is a rough estimate and its real value should be measured from experiments. Since in the current work we are primarily concerned about the range of the parameters covered in some worst scenarios, it is reasonable to set 30,000 W/m^2 K as the upper limit of the coefficient of convection in the discussions.

2.7. Radiative cooling

In addition to convection, radiation may also affect the overall heat dissipation rate due to the high operating temperatures. The Stefan–Boltzmann law denotes

$$\mathbf{Q}_{rad} = \varepsilon\sigma(\mathbf{T}^4 - \mathbf{T}_\infty^4) \tag{29}$$

where ε is the surface emissivity and σ is the Stefan–Boltzmann constant. By differentiating the above equation, one may obtain a linearized radiation rate in the perturbation form

$$\mathbf{Q}_{rad} = 4\varepsilon\sigma\mathbf{T}_{avg}^3\mathbf{\Theta} \tag{30}$$

where T_{avg} is the average surface temperature. If this equation is combined with the convective heat rate, then the radiation heat transfer can be incorporated into the above formulation by

augmenting the convection coefficient with a correction term as following

$$h' = h + \Delta h \tag{31}$$

where

$$\Delta h = 4\varepsilon\sigma \mathbf{T}_{avg}^3 \tag{32}$$

with this correction, both radiation and convection can be incorporated into the overall heat transfer coefficient h.

2.8. Mesh considerations

In the TEI problem involving two sliding bodies, it is well characterized that there is a *thermal skin* in the vicinity of the contact surface of the poor conductor [7]. This is because the disturbance moves relatively slowly over the good conductor but with a speed close to the sliding speed over the friction material. As a result, any given location on the surface of the friction material experiences an oscillatory temperature at a very high frequency, which in turn generates very short heat waves in a direction perpendicular to the surface. To describe the temperature field adequately it is essential to use a graded mesh in this region to ensure that enough elements are generated in the thermal skin layer. In reference [7] the criteria were developed to determine the mesh refinement for this purpose and those criteria are equally applicable to the present problem. Based on these criteria a mesh bias ratio of 2.0 has been used in the friction layer and 1.5 in the metal layer, with more elements created towards the sliding interface.

3. Computational results

3.1. Model parameters

The parameters used in the finite element analyses were obtained from the literature [1] and they are tabulated in Tables 1 and 2. These parameters reflect the realistic clutch and brake systems used in the automotive industry, although the actual values may vary in different applications. Note that in the brake model, a 1/6 coverage, or 60 degrees of pad section was assumed and therefore the equivalent coefficient of friction is $0.4/6 = 0.0667$ for the full coverage. Both clutch and brake models assume an antisymmetric boundary condition along the midplane of the rotor (Fig. 1) because the antisymmetric modes have proved more susceptible to TEI based upon the previous researches. Therefore a half thickness has been used for the metal layer (steel or cast iron) in both models. In all the following discussions, the sliding velocity V is normalized as V^* defined by

$$V^* = \frac{Va}{k_m} \tag{33}$$

where a and k_m are the half thickness and the thermal diffusivity of the metal disk, respectively. This is consistent with the dimensionless velocity defined in Lee and Barber's work [3]. The

Table 1
Parameters used in the clutch model.

	Steel disk	Friction layer
Young's modulus, E (GPa)	200	0.11
Poisson's ratio, v	0.30	0.25
Thermal expansion coefficient, α (K^{-1})	1.2×10^{-5}	1.4×10^{-5}
Thermal conductivity, K (W m^{-1} K^{-1})	42	0.22
Thermal diffusivity, k (mm^2 s^{-1})	11.9	0.122
Thickness (mm)	1.375	0.673
Coefficient of friction	0.12	0.12

Table 2
Parameters used in the brake model.

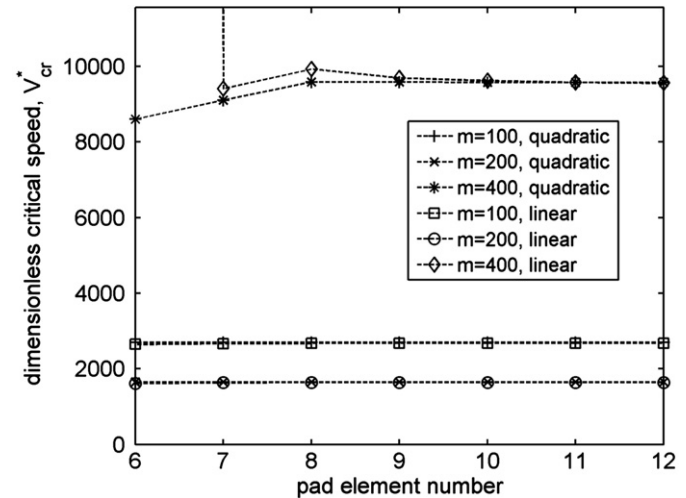|	Cast iron disk	Friction layer
Young's modulus, E (GPa)	112.4	2.03
Poisson's ratio, v	0.25	0.35
Coefficient of thermal expansion, α (K^{-1})	1.325×10^{-5}	3.0×10^{-5}
Thermal conductivity, K (W m^{-1} K^{-1})	57	0.93
Thermal diffusivity, k (mm^2 s^{-1})	17.2	0.522
Thickness (mm)	14	10
Coefficient of friction	0.4	0.4

Fig. 2. Critical speed as a function of the element number in the friction disk (pad) of the clutch model.

convection coefficient h is normalized as Bi, i.e. the *Biot number* defined by

$$Bi = \frac{ha}{K_m} \tag{34}$$

where K_m is the thermal conductivity of the metal disk. A convection coefficient of 10,000 W/m^2 K is equivalent to a *Biot number* of 0.3274 in the clutch model or 2.456 in the brake model being discussed.

3.2. Comparisons between linear and quadratic elements

A series of convergence tests were attempted to study the effect of the mesh size on the result. Fig. 2 shows the critical speed as a function of the element number in the friction layer, for three different wave numbers $m = 100$, 200 and 400. A comparison has been made between the two finite element types: linear elements and quadratic elements. The element number in the friction layer varies between 6 and 12 in both cases. It can be seen that at lower wave numbers $m = 100$ or 200, there is virtually no difference between the two element types whereas the difference is appreciable when $m = 400$ and when the element number is less than 10. The main reason is that the temperature field oscillates at a higher frequency for shorter wavelengths, thus requiring a finer mesh to capture the variations in the temperature field. In this case use of quadratic elements has an obvious advantage over the linear elements. In fact, when the element number is less than seven, the solution using the linear elements diverges quickly. In contrast the model using the quadratic elements is still capable of yielding an acceptable solution. An inspection on the results shown in Fig. 3, where the critical speed is expressed as a function of the element number in the steel
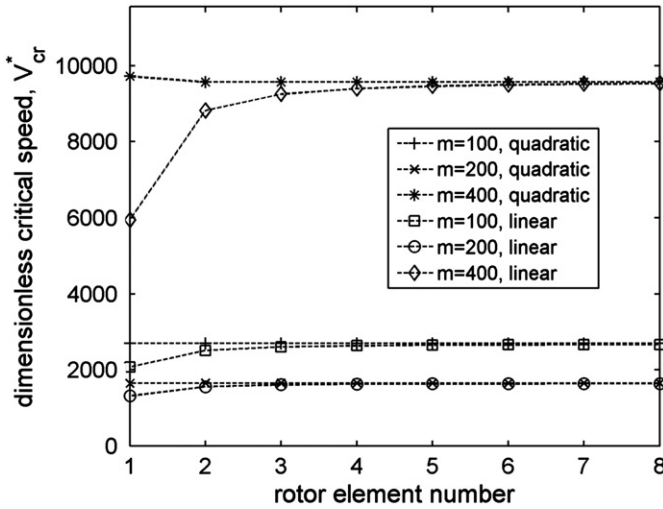
Fig. 3. Critical speed as a function of the element number in the steel disk (rotor) of the clutch model.

layer, has revealed a similar trend. That is, the linear elements lead to a solution very close to that obtained from the quadratic elements at lower wave numbers, yet they exhibit a significant difference for higher wave numbers in terms of the convergence speed. For the quadratic elements, a surprisingly accurate result, which deviates merely about 1.5% from the convergent solution at $m=400$, can be obtained using only one element across the thickness of the metal rotor. Although the computational efficiency is generally not a major concern in the current problem, it has a significant importance for higher dimensions. The previous work showed that the computational time for a three dimensional system is very sensitive to the total DOFs (degrees-of-freedom). The results presented in Figs. 2 and 3 imply that a fine mesh is not always necessary and a fairly coarse mesh in the metal disk using quadratic elements, for example, is a potential solution to improve the numerical efficiency for problems defined in higher dimensions.

3.3. Effect of convective cooling on temperature growth rate

To investigate the effect of the convection heat transfer coefficient, the exponential temperature growth rate b has been plotted against the sliding velocity V in the clutch model using three different values of the convection coefficient when m was chosen as 200 (Fig. 4). This value of m is approximately located at the lowest critical sliding speed among all wave numbers. The results are presented in an effort to demonstrate the transition of the system from a stable mode to an unstable mode as the sliding speed increases. When the convection heat dissipation is absent, the growth rate is negative at a low sliding speed showing that the temperature perturbation decays with time. When V^* is close to 1652, the growth rate b approaches zero, which corresponds to a transition stage where the perturbation starts to grow. Beyond that point, the growth rate becomes positive, representing a thermoelastically unstable condition. Therefore this particular location on the horizontal axis corresponds to the critical speed of the system. It appears that the relationship between the growth rate and the sliding speed is almost linear in the unstable region. In the stable region, on the other hand, it starts with a nonlinear profile at a low speed, but becomes nearly linear in the remaining part. This is consistent with the prior results reported in the literature [8]. The addition of a nonzero convection coefficient has apparently shifted the curve to the right side, resulting in an increase in the critical speed, but it does not alter
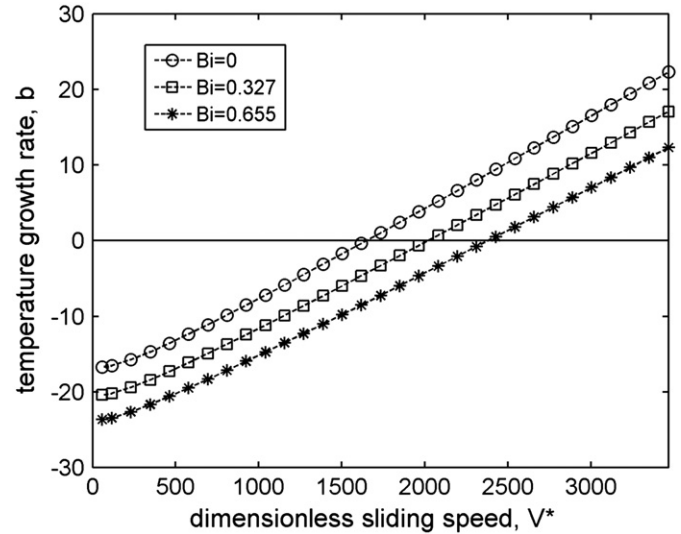

Fig. 4. Temperature growth rate as a function of the sliding velocity in the clutch model for $m=200$.
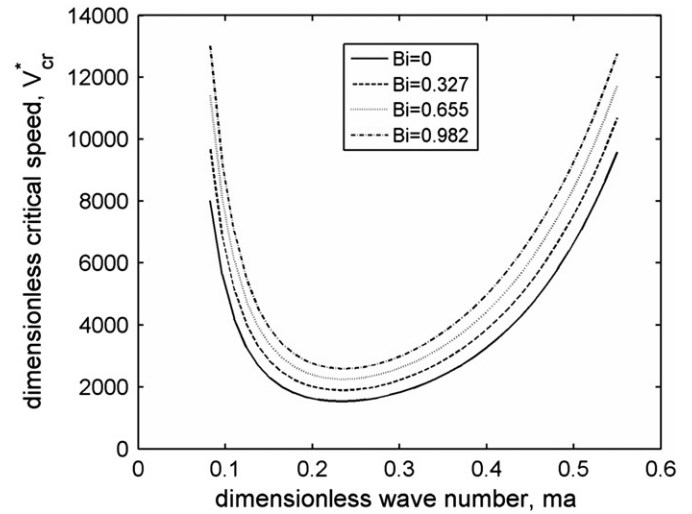

Fig. 5. Critical speed as a function of the wave number in the clutch model.

the overall shape of the curve, i.e. it begins with a slightly nonlinear region and smoothly transits to a nearly linear profile. The three curves shown in Fig. 4 are approximately parallel to each other. It is estimated that an increase of the convection coefficient by 10,000 W/m² K ($Bi=0.327$) approximately raises the critical speed by $\Delta V^*=347$ in the current clutch model, or around 20% of the value without convection.

3.4. Effect of convective cooling on critical speed

Further, the critical speed was obtained as a function of the wave number m under a set of different convection coefficients ranging $h=0$, 10,000, 20,000 and 30,000 W/m² K (i.e. $Bi=0$, 0.327, 0.655 and 0.982) for the clutch problem, as shown in Fig. 5. When the convection heat dissipation is absent, the lowest critical speed is located at the dimensionless wave number around $ma=0.234$ with a value of $V^*=1537$. This is very close to the critical value 1652 obtained at $m=200$ (i.e. $ma=0.275$) shown in Fig. 4. The entire curve has been raised by the addition of thermal convection, but the overall shape of the curve does not change much, with the lowest critical speed maintained at the same location

of m. Presented in Fig. 6 is the similar results for the brake model with the lowest critical speed occurring at $ma=0.532$. Unlike the clutch results shown in the previous figure, the V–m curve seems to keep changing its shape as the convection coefficient increases. A close look at the data, however, has shown that the wave number corresponding to the lowest critical speed does not change as the convection coefficient increases. An immediate implication of the result is that there should not be any change in the dominant mode shapes at any given sliding speed, either. That is, the eigenmodes change their b values at the same rate as the convection coefficient changes.

Fig. 7 shows how the minimum critical speed varies with the convection coefficient under three representative wave numbers $m=100$, 200 and 400 in the clutch model. It can be seen that the critical speed is almost a linear function of the convection coefficient. When $m=100$, the critical speed changes from $V^*=2692$ at $h=0$ ($Bi=0$) to $V^*=4553$ at $h=30,000$ W/m² K ($Bi=0.982$), with an approximately 69% increase. In terms of percentage change this is the maximum among the three curves in Fig. 7. For example, at $m=400$ the change in the critical speed is from $V^*=9579$ to 12,756, with an approximately 33.2% increase. A similar conclusion can be
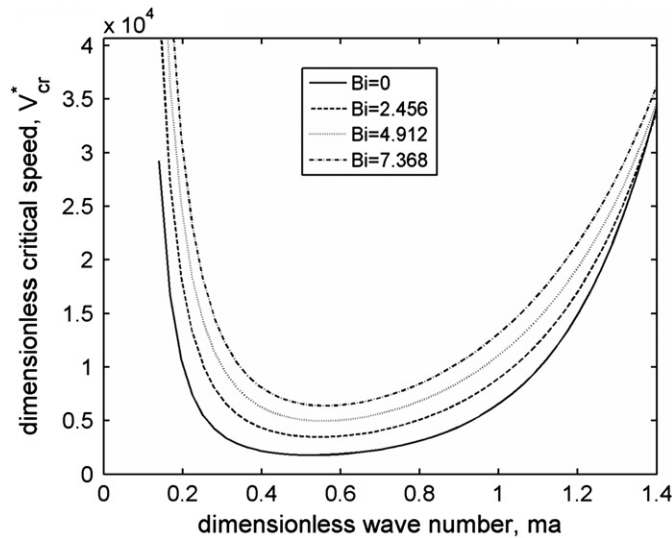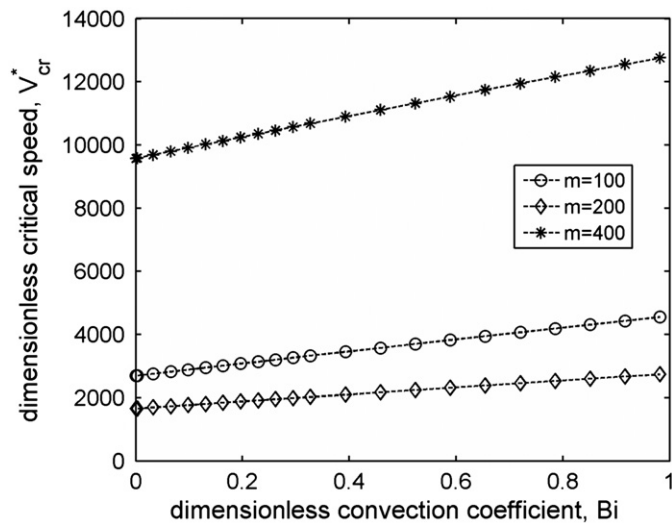

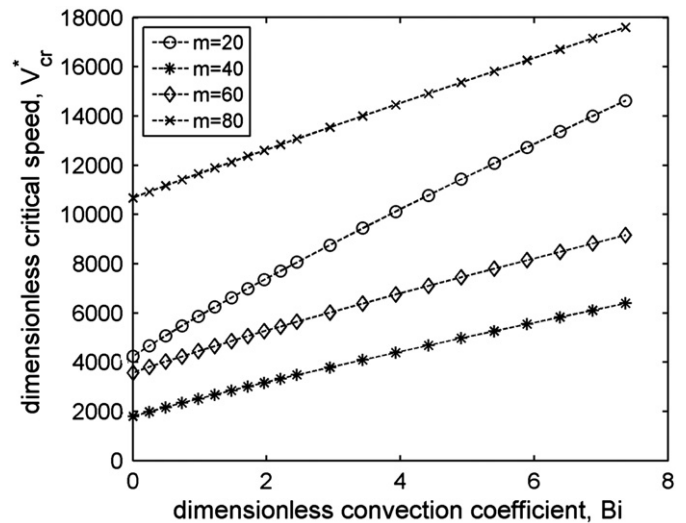**Fig. 8.** Minimum critical speed as a function of the convection coefficient in the brake model.

drawn from the brake model, as shown in Fig. 8. The four curves representing $m=20$, 40, 60 and 80 shown in the figure have different slopes, implying different degrees of dependence of the critical speed on the convection rate. This dependence is the strongest at the lowest wave number ($m=20$) among the four. Compared to the clutch model this time the difference is even more drastic: the critical $V^*=4233$ at $h=0$ ($Bi=0$) versus $V^*=14,651$ at $h=30,000$ W/m² K ($Bi=7.368$). That is equivalent to an increase of 246%. At $m=40$ where the critical speed almost reaches the valley in the V–m curve, it is found that the critical speed changes from $V^*=1799$ to 6390 when h varies from 0 to 30,000 W/m² K ($Bi=0$–7.368), or an approximately increase of 255%. In fact, even with $h=1000$ W/m² K ($Bi=0.246$), the increase in the critical speed for $m=40$ is as much as 10.4% in comparison with the result at $h=0$, which is a non-negligible difference. On the other hand, at $m=80$, percentage change of the critical speed is close to 65% for the full range of the convection coefficient. Therefore it has been seen that the effect of convection coefficient is more significant for the eigenmodes with shorter wavelengths.

3.5. Effect of radiation cooling

Fig. 9 shows the equivalent coefficient Δh as a function of the surface temperature under the different values of thermal emissivity ε ranging from 0.2 to 1.0, based on Eq. (32). Clearly, the thermal radiation rate is much less significant than convection. Even with $\varepsilon=1$ and $T_{avg}=1500$ K, the equivalent coefficient Δh is still below 1000 W/m² K (i.e. $\Delta Bi < 0.0327$ in the clutch model or $\Delta Bi < 0.246$ in the brake model), which is an order of magnitude less than the convection coefficients of common liquids. With a practical disk surface temperature around 500 °C and a realistic thermal emissivity of the material, it is found that Δh is well below 100 W/m² K. Therefore the effect of thermal radiation on TEI is usually negligible compared to convective cooling.

Fig. 6. Critical speed as a function of the wave number in the brake model.

4. Conclusions

A finite element method is implemented to investigate the effect of convective cooling on the stability boundaries of thermo-elastic instability in a couple of representative brake and clutch systems. By adding a negative term representing the convective heat dissipation to the frictional heat generation rate, convective cooling
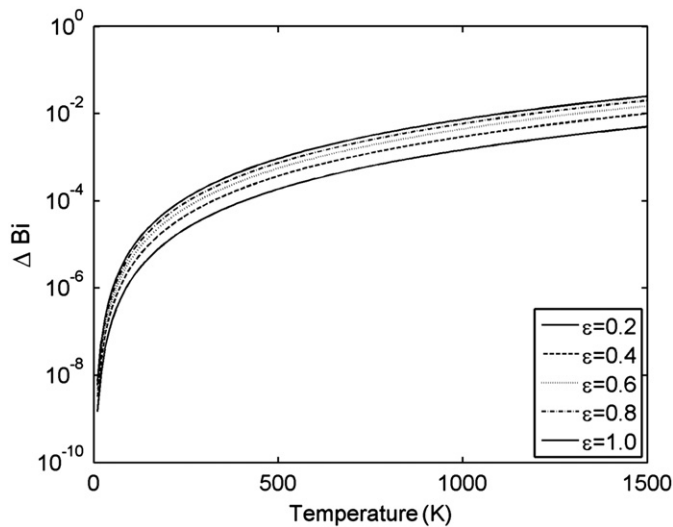

Fig. 7. Minimum critical speed as a function of the convection coefficient in the clutch model.

Fig. 9. Equivalent *Biot number* for thermal radiation as a function of the surface temperature.

is successfully incorporated into the finite element formulation. This is analogous to a system with a reduced frictional heat rate, and therefore can stabilize the thermal–mechanical process. As a consequence it has been found that the previous analyses on TEI typically overestimated the critical sliding speeds. Liquid cooling such as water and lubricants removes heat at a much faster rate than gases such as air, and therefore affect the system stability more significantly. The parametric studies have shown that the critical speed in some cases where the system is cooled by water can be three times as high as the value without convective cooling. However, the wave number corresponding to the lowest critical speed is nearly independent of the convection effect. This implies that the dominant mode pattern at a given sliding speed remains unchanged as well. Further, in comparison with the linear elements, the quadratic elements are capable of accurately capturing the oscillatory patterns of the temperature in the vicinity of the friction interface, and therefore provide a better numerical accuracy with the same computational effort. Although using quadratic elements is

not necessary in lower dimensions, it will potentially permit efficient solutions for problems defined in higher dimensions.

References

[1] Y.B. Yi, Finite Element Analysis of Frictionally Excited Thermoelastic Instabilities in Automotive Brakes and Clutches, Ph.D. Dissertation, University of Michigan, 2001.
[2] J.R. Barber, Thermoelastic instabilities in the sliding of conforming solids, Proceedings of the Royal Society of London Series A 312 (1969) 381–394.
[3] K.J. Lee, J.R. Barber, Frictionally-excited thermoelastic instability in automotive disk brakes, ASME Journal of Tribology 115 (1993) 607–614.
[4] L. Afferrante, M. Ciavarella, J.R. Barber, Thermoelastodynamic instability (TEDI): a new mechanism for sliding instability, Proceedings of the Royal Society of London Series A 462 (2006) 2161–2176.
[5] S. Du, P. Zagrodzki, J.R. Barber, G.M. Hulbert, Finite element analysis of frictionally-excited thermoelastic instability, Journal of Thermal Stresses 20 (1997) 185–201.
[6] P. Zagrodzki, Thermoelastic instability in friction clutches and brakes—transient modal analysis revealing mechanisms of excitation of unstable modes, International Journal of Solids and Structures 46 (2009) 2463–2476.
[7] Y.B. Yi, J.R. Barber, P. Zagrodzki, Eigenvalue solution of thermoelastic instability problems using Fourier reduction, Proceedings of the Royal Society of London Series A 456 (2000) 2799–2821.
[8] R.A. Burton, V. Nerlikar, S.R. Kilaparti, Thermoelastic instability in a seal-like configuration, Wear 24 (1973) 177–188.
[9] Y.B. Yi, S. Du, J.R. Barber, J.W. Fash, Effect of geometry on thermoelastic instability in disk brakes and clutches, ASME Journal of Tribology 121 (1999) 661–666.
[10] D.L. Hartsock, J.W. Fash, Effect of pad/caliper stiffness, pad thickness, and pad length on thermoelastic instability in disk brakes, ASME Journal of Tribology 122 (2000) 511–518.
[11] S.W. Lee, Y.H. Jang, Effect of functionally graded material on frictionally excited thermoelastic instability, Wear 266 (2009) 139–146.
[12] P. Decuzzi, G. Demelio, The effect of material properties on the thermoelastic stability of sliding systems, Wear 252 (2002) 311–321.
[13] S. Ramousse, J.W. Hoj, O.T. Sorensen, Thermal characterisation of brake pads, Journal of Thermal Analysis and Calorimetry 64 (2001) 933–943.
[14] J.H. Choi, J.H. Han, I. Lee, Transient analysis of thermoelastic contact behaviors in composite multidisk brakes, Journal of Thermal Stresses 27 (2004) 1149–1167.
[15] Anonymous, AquaMaKKs clutches and brakes stop corrosion, with new composite water jackets that allow cooling with salt water, Anti-Corrosion Methods and Materials 58 (2011) 155–155.
[16] T.L. Bergman, A.S. Lavine, F.P. Incropera, D.P. DeWitt, Fundamentals of Heat and Mass Transfer, 7th ed., John Wiley & Sons, Inc, Hoboken, NJ, 2007.
[17] P. Zagrodzki, Effect of Newtonian Cooling on Frictionally-Excited Thermoelastic Instability in Wet Friction Clutches and Brakes, In: Proceedings of the 8th International Congress on Thermal Stresses, 2009, June 1–4, 2009, Urbana-Champaign, USA.
[18] P. Payvar, Laminar heat-transfer in the oil groove of a wet clutch, International Journal of Heat and Mass Transfer 34 (1991) 1791–1798.